1 Introduction

Reasoning about actions and change is one of the central and most active reasearch areas in Artificial Intelligence. Actions can be represented by action verbs, such as to take, to give, to open, to marry, ... Actions describe dynamic world changes and the semantics of action verbs (and of actions) has to take into account these changes. For an example, when the action "take(john,ball)" occurs in a situation where a red ball is on the floor in a room with a white table and four chairs, the facts *on(floor,ball)* and *not holding(john,ball)* will change. *on(floor,ball)* and *not holding(john,ball)* will become false and *holding(john,ball)* will become true. On the other hand, the facts *color(ball,red), in(room,table), color(table,white), ...* and most others will **not** change. The action "take(john,ball)" has the direct, obvious result *holding(john,ball)*, but has probably also the indirect result *not on(floor,ball)*. While it is rather easy to exhibit the direct results of actions and to conceive action theories which can take into account these direct world changes, it is much more difficult to take into account the indirect changes and the facts which do not change. The problem of persistency of facts has been identified first in 1969 by John MacCarthy and Patrick Hayes [19] as the *frame problem*. The problem of deriving indirect changes has been identified first in 1986 by Henry Kautz [12] as the *ramification problem*. The third fundamental problem related to actions and change is the *qualification problem* and concerns the complete formulation of all preconditions necessary for the execution of an action. In this paper we will only be concerned with solutions to the two former problems, the *frame and ramification problems*.

During the last thirty years, a very great number and a very great variety of logic based approaches to reasoning about action and change have been proposed. Despite of great differences, these approaches have many things in common:

- Actions are most time specified by a pair of formulas, π, ρ, where π represents the *preconditions* necessary for executing the action and ρ represents the results of the action. This means that an action can be executed when its preconditions are true and subsequently, its results will become true.
- Actions occur in a situation (or a state) of the world. Consequently in action theories it is possible to represent actual, future and eventually past world states by formulas.
- Most high level action theories allow to specify so called "world laws" (or "general laws", or "constraints"). These are formulas which should be true in all situations and never can change.

It is obviously not possible to solve the frame problem by explicitely enumerating persistency axioms. This is impossible for two reasons: first, such an enumeration would include too much persistency rules; second, the needed persistency rules depend on the situation in which an action occurs. For an example, if we push a table from one place to another, the changes and persistencies depend on what is actually on the table. If there are objects on the table, these objects change also their place. If these objects are not on the table, but somewhere else, their place will not change. Both, the frame and the ramification problem depend on the context of the world and they are interdependent.

The most obvious solution to the frame problem consists in minimizing the changes in the world during the execution of an action. The most obvious solution to the ramification problem consists in taking into account all changes indirectly caused by direct changes.

Let us consider the following example, which has been treated by several authors and which will serve as one benchmark example.

Example 1 *Let us consider a suitcase which is closed by two latches. The atoms* $up1$ *and* $up2$ *and* $open$ *represent this example:*
$up1$ *means that latch 1 is up.*
$up2$ *means that latch 2 is up.*
$open$ *means that the suitcase is open.*

This scenario is characterized by one constraint

$$(W) \quad up1 \wedge up2 \rightarrow open$$

expressing that the suitcase is open whenever the two latches are up.

Consider an initial situation

$$(S) \quad \neg up1 \wedge up2 \wedge \neg open$$

There are two possible actions

$$\begin{array}{rl} flip1 = & (\neg up1, up1) \;\; or \;\; (up1, \neg up1) \\ flip2 = & (\neg up2, up2) \;\; or \;\; (up2, \neg up2) \end{array}$$

In (S), the precondition for $flip1$, $\neg up1$, is verified, hence the action can be executed, yielding the result $up1$.
What happens to $up2$ and to $\neg open$? Do they persist? Which one persists?

Consider the following two possibilities:

1. $up2$ persists after $flip1$. Then, in the new situation $up1$ and $up2$ are both true and the constraint (W) $up1 \wedge up2 \rightarrow open$, which is true in all situations, allows to derive $open$. Therefore, $\neg open$, which was true in the initial situation (S), does not persist. This means that $open$ changes its value as a ramification effect of $flip1$.

2. $\neg open$ persists after $flip1$. Then, in the new situation $up1$ and $\neg open$ are both true and the constraint (equivalent to (W)) $up1 \wedge \neg open \rightarrow \neg up2$, which is true in all situations, allows to derive $\neg up2$. Therefore, $up2$, which was true in the initial situation (S), does not persist. This means that $up2$ changes its value as a ramification effect of $flip1$.

Obviously, most poeple would prefer the first scenario, where $open$ changes. There is no reason whatsoever that the second latch gets down when one flips the first latch. They are independent!

The unintuitive change is due to the formulation of the constraint which uses classical implication. The constraint (W) as expressed so far is equivalent to the following laws:

$$\begin{array}{ll} (W) & up1 \wedge up2 \rightarrow open \\ & \neg up1 \vee \neg up2 \vee open \\ & up1 \wedge \neg open \rightarrow \neg up2 \\ & up2 \wedge \neg open \rightarrow \neg up1 \\ & \ldots \end{array}$$

Hence one possibility to prevent the application of these laws is to block the equivalences!

One way to achieve this is to replace the classical implication $\rightarrow$ by another **causal** implcation $CAUSES$.

So, one can propose to replace the law (W) by a new law

$$(C) \quad up1 \wedge up2 \quad \text{CAUSES} \quad open$$

Then the following question arises: How can causal implication be formalized? Obviously, causal implication is weaker than classical implication. For example contraposition, which allows to apply the law (W) in the different ways, should not hold for causal implication. Subsequently, we will discuss properties of causal implication.

2 Causal implication

Some properties of classical implication should not hold for causal implication. Other properties should hold.

We discuss criteria representing properties which might be desirable for causal implication, such as reasoning by case, to give an example. But there are also criteria representing properties which causal implication should not have. The criteria are discussed independently on the language level on which causal implication (or the causality relation) is represented.

2.1 Criteria for causality

We will try to situate each formalism in relation to the following criteria, considered as meaningful for causality.

1. Monotonicity
   Classical implication " $\rightarrow$" is monotonic. This means that from $\phi \rightarrow \psi$, we can conclude $\phi \wedge \chi \rightarrow \psi$. But the same is not so obvious for causal implication. If in the introductory example, $latch_1$ and $latch_2$ are both *up*, then the suitcase will be *open*. If both latches are *up* and the suitcase is closed by a leather belt, then the suitcase is not *open*. We think, that causality is not monotonic. To give another example: If it rains, then John will get wet clothes. If it rains and John has an umbrella, he will *not* get wet clothes. So, we have

   rain CAUSES *wet* but not necessarily $rain \wedge umbrella$ CAUSES *wet*

   Obviously, many examples of this type may be found. The causal theories which we consider are typically for issues of commonsense reasoning. Causal implication which is monotonic cannot cope with exceptions. It is then not possible to handle situations where an event or fact causes another event or fact, but does not cause it any more in the presence of other facts. The causation of a fact by another fact may be disturbed by the arrival of new facts.

2. Transitivity
   The classical implication is transitive. This means that from $\phi \rightarrow \psi$ and $\psi \rightarrow \chi$, we conclude $\phi \rightarrow \chi$. Arguments can be found for supporting that causality is transitive. For an example if ten blocks are on a table one next to the other, pushing the leftmost block to the right may cause all the blocks to be pushed to the right since each block pushes the next one. But it is equally possible to find examples which suggest that causality is not transitive. Consider the following sample scenario:
   John is a broker and holds a very big stock. If he is in a room and

somebody puts the television on the aeration duct, then the room becomes stuffy. If he is in a stuffy room, he gets a headacke. If he has got a headache, then he can get depressive. If he is depressive, then he will sell all his stock. If he sells his stock, this causes a minicrash! But can we conclude that putting the television on the aeration duct will cause a minicrash? This kind of example shows, that causality is certain not more transitive, if we construct a causal chain which is sufficiently long; in this case, the cause and the consequences just will be too far away one from each other.

As a consequence of these considerations, we think that transitivity should neither be allowed nor forbidden as property of causal implication.

3. Contraposition

   Classical implication is contrapositive. This means that from $\phi \rightarrow \psi$ is true, we can conclude that $\neg\psi \rightarrow \neg\phi$ is true. Causality is certainly *not* contrapositive, as illustrates the following example.
   "If the sun is shining, I will go to the beach" might be true. But it is possible that I do not go to the beach, however the sun is shining. Hence the contraposition "if I do not go to the beach, then the sun is not shining" might not be true!!

4. Reflexivity
   Classical implication is reflexive, we have that $\phi \rightarrow \phi$ is true for any ϕ. But causality should not be reflexive. Nothing can cause itself!

5. Conjunction of preconditions
   For classical implication, if $\phi \wedge \pi \rightarrow \psi$ is true, then either $\phi \rightarrow \psi$ or $\pi \rightarrow \psi$ is true. This does not hold for causal implication as shows the following example:
   If I am in the bain tub and I use the hair dryer, this causes a danger for my life. But neither one of these facts alone causes a danger for my life. If somebody is in his bain tub, normally there is no danger for his life, neither if he uses a hair dryer.

   Hence we can have

   A AND B CAUSES C but neither A CAUSES C nor B CAUSES C

   The reason is that A and B interact causing TOGETHER C. And logical connective $\wedge$ cannot represent alone this interaction.

   The following are properties which a causality relation should have.

6. Conjunction
   Just as for implication, if a fact causes two other facts, it should cause their conjunction. Thus we should have

   $$if\ \phi \text{ CAUSES } \psi \text{ and } \phi \text{ CAUSES } \pi \text{ then } \phi \text{ CAUSES } \psi \wedge \pi$$

   For an example, from "stuffy(room) causes hot" and "stuffy(room) causes thursty", we infer "stuffy(room) causes hot and thursty"!

7. Reasoning by case
   A causal relation should allow for reasoning by case. We think $\phi \vee \pi$ CAUSES ψ should be true, whenever ϕ CAUSES ψ and π CAUSES ψ are true.

For an example, if John's right arm is broken, he cannot work any more. If John's left arm is broken, he cannot work neither. We know that either his left or his right arm is broken and we want to conclude that he cannot work any more.

8. Right weakening RW
We think that a formula should cause all logical consequences of the formulas it causes, i.e. if ϕ CAUSES ψ and $\models \psi \to \chi$, then ϕ CAUSES χ. This is a specific form of the well known rule RCK which holds for all systems of conditional logic. This rule holds also for cumulative and preferential logics and has been used (and called "right weakening") by Sarit Kraus, Daniel Lehmann and Menachem Magidor in [13, 14].

9. Left logical equivalence LLE
Classically equivalent assertions should have the same causal consequences. This is the well known rule RCEA of conditional logic. A more special form of this rule has been used (and called "left logical equivalence") for cumulative and preferential logics [13, 14].
If ϕ CAUSES ψ and $\models \phi \leftrightarrow \chi$ then χ CAUSES ψ.

2.2 Representation of Causality

We have not yet specified on which level we will consider causality. We have discussed "causal implication" in a very general way without fixing more precisely, what this is. Remember, that a *logic* is an inference relation on formulas, which are expressions of a *language*. Hence, given a language and the set of formulas of this language, FOR, a logic is a subset of $FOR \times FOR$. In addition, in classical logic, there is one connective $\to$, that represents implication on the language level. And it is well known, that for classical logic, there is a one-to-one correspondence between the inference relation, denoted $\vdash$, and the implication, denoted $\to$. This means, that the deduction theorem and its converse hold:

$$\phi \vdash \psi \text{ if and only if } \vdash \phi \to \psi$$

So, we might ask the question, how causality can be or should be represented. There are several possibilities to represent causality and we can find in the literature many different ways.

- Causality could be represented as an inference relation on *classical formulas*. This means, that a special relation CAUSES is defined,

$$CAUSES \subset FOR \times FOR$$

  This new inference relation can be characterized by axioms and derivation rules.

- Another approach is to remain completely in the framework of classical logic and to express causality by a new non-logical predicate *Caused*. Lin [17] (and many others) adopts this solution. He defines a new classical predicate $Caused(f, v, s)$, meaning, that the fluent f is caused to get the truth value v in state s.

- Another approach consists in using a new implication operator on the language level. One gets of course very close to conditional logics with this approach. Traditionally, one interpretation (or understanding) of the conditional has been to consider it as a causal implication [11, 15].

- Finally, there have also been proposals to define causality as a unary modal operator ©. ©ϕ then is read to mean that ϕ is caused. This approach has been used by Hector Geffner [6], Laura Giordano, Alberto Martelli and Camilla Schwind [7] and by Hudson Turner [23].

So far a representation level has been chosen, the next step is to consider how the causal inference should "work", i.e. how causality should be used to obtain conclusions from a set of formulas and causal implication rules and assertions. According to the representation level chosen, there will be the following possible mechanisms.

- If causality is considered as an inference relation, it defines a logic, is ruled by axioms and inference rules and is characterized by a semantics, without necessarily extending the basic object language.
- If causality is formulated in the framework of a classical language by introduction of a special new non-logical predicate, there will be special nonlogical axioms for specifying properties of this predicate.
- A causal or modal operator, defines a new logical language, which is an extension of the classical language and is ruled by logical axioms and inference rules and is characterized by a notion of semantics.

3 Approaches to Actions and Causality

In this part, we will present a number of approaches to actions and change which have been proposed in the last years. Without being exhaustive, we have tried to choose approaches which seem to be typical for the choice of their representation level. Our goal has not been to present all approaches but to present typical approaches for a certain choice of language and logic. Moreover, we restricted ourselves to approaches which are concerned with actions and causality and did not consider approaches which are merely concerned with causality alone.

One remark should be made about other non-logical approaches to causality which exist in the domain of probability and Bayesian trees and network. We did not consider these approaches because we wanted to restrict our overview to logic based approaches. And it seems impossible to apply the observations we applied to these kind of approaches. If causality is not conceived as a special type of "weak implication", then it is not possible to discuss properties related to this concept. Even with the logic based approaches, we will present here, we will see that sometimes their expressivity is restricted and some properties cannot hold for this reason.

We will discuss in greater detail four approaches.

1. Fangzen Lin in [17, 18] presents a theory for actions and causality which is based on classical predicate logic with circumscription. The causality relation $f\ CAUSES\ g$ is represented by a formula $Holds(f, s) \rightarrow caused(g, v, s)$, meaning "if f is true in the situation s, then g is caused to be v, where $v \in \{true, false\}$, in the situation s.
2. Norman McCain and Hudson Turner in [20, 21], represent causality by means of a conditional type operator $\Rightarrow$. $\phi\ CAUSES\ \psi$ is represented by a formula $\phi \Rightarrow \psi$. A special semantics is defined for specifying what are the consequences of a set of causal rules of this form. Lifschitz in [16] generalizes this notion of causality (for first-order formulas ϕ and

ψ. One extension of the action description language $\mathcal{A}$ [9] is based on this theory.

3. Laura Giordano, Alberto Martelli and Camilla Schwind in [7] use a unary modal operator © for expressing that a formula is caused. ϕ CAUSES ψ is expressed by $\phi \rightarrow$ ©ψ, meaning "if ϕ holds, then ψ is caused".

4. Michael Thielscher in [22] uses causal rules of the form $l\ CAUSES\ l'\ if\ \bigwedge_i l_i$, where l, l' and l_i are literals. These rules are generated automatically from clauses by means of *influence information.*

3.1 Lin's approach to Actions and Causality

Fangzen Lin in [17, 18] uses a sorted first order language with a circumscription policy for representing action theories. He introduces a non logical predicate "Caused(f,v,s)", meaning that fluent f is caused to have the truth value v in the situation s. The basic axioms for $Caused$ are

$$C1 \quad Caused(f, true, s) \rightarrow Holds(f, s)$$

and

$$C2 \quad Caused(f, false, s) \rightarrow \neg Holds(f, s)$$

Action laws are expressed by formulas of the form

$$(A)\ Poss(a, s) \rightarrow (Holds(\phi, s) \rightarrow Caused(f, v, do(a, s)))$$

where f is a fluent name and $v \in \{true, false\}$. If a is possible in state s and ϕ holds in s, then f is caused to become v in $do(a, s)$ denoting the state resulting from executing a in s. The predicate $Caused$ is also used for formulating causal relationships within domain constraints. For example, the domain constraint of the suitcase Example 1 is then $Holds(up(1), s) \wedge Holds(up(2), s) \rightarrow Caused(open, true, s)$. $Caused$ is circumscribed in the theory consisting of constraints and action laws. This yields a first-order theory where a fluent f is $Caused$ (to admit the truth value v), whenever f is caused to be v as an immediate or ramification effect of an action. Frame laws (F) expressing that a fluent persists as long as it is not *caused* to change its value are added to the theory *after* the minimization of *caused.*

$$(F)\quad \neg\exists v\ Caused(f, v, do(a, s)) \rightarrow (Holds(f, s) \leftrightarrow Holds(f, do(a, s)))$$

(F) expresses that f persists from state s to state $do(a, s)$ whenever it is not caused to get a truth value v by occurrence of an action a in state s. The *Caused* predicate is used in order to control the persistency of fluents: only fluents which are caused to have a truth value v are allowed to change value (and for this reason the *Caused* predicate has to occur also in action laws) and only fluents which are *not* caused to have a truth value v are allowed to persist.

Let us now consider the suitcase example expressed in Lin's theory.

Example 2 *The theory T consists of the general causal laws (C), of domain laws (W) and of action laws (A)*

$$\begin{array}{ll}
C & Caused(f, true, s) \rightarrow Holds(f, s) \\
 & Caused(f, false, s) \rightarrow \neg Holds(f, s) \\
W & Holds(up(1), s) \wedge Holds(up(2), s) \rightarrow Caused(open, true, s) \\
A & Poss(flip(i), s) \rightarrow (\neg Holds(up(i), s) \rightarrow Caused(up(i), true, do(flip(i), s))) \quad i = 1, 2 \\
 & Poss(flip(i), s) \rightarrow (Holds(up(i), s) \rightarrow Caused(up(i), false, do(flip(i), s))) \quad i = 1, 2
\end{array}$$

Given an initial situation s_0, the following can be derived

(1)	$\neg Holds(up(1), s_0) \wedge Holds(up(2), s_0) \wedge \neg Holds(open, s_0)$	initial situation
(2)	$Holds(up(1), do(flip(1), s_0))$	from (1) by application of the action law for flip(1)
(3)	$\neg\exists\ v\ Caused(up(2), v, do(flip(1), s_0))$	by circumscribing Caused
(4)	$Holds(up(2), do(flip(1), s_0))$	from (3) and (1) by the frame law (F)
(5)	$Caused(open, true, do(flip(1), s_0))$	from (4) and (2) by the causal law (W)
(6)	$\exists v Caused(open, v, do(flip(1), s_0))$	from (5) preventing open to persist over flip(1)

The example works well and gives the desired results.

We now study the behaviour of Lin's theory with respect to the properties we exhibited in the last chapter. First, let us observe that causality cannot be expressed for arbitrary formulas, but only for atoms. So, Lin can merely express "p CAUSES q", for atoms p and q, but not ϕ CAUSES ψ for formulas ϕ and ψ. Hence, it is impossible to consider properties, which require arbitrary non atomic formulae. While it is admitted that $Holds(p\,c\,q, s)$ means $Holds(p, s)\ c\ Holds(q, s)$, for any binary logical connector c, and $Holds(\neg p, s)$ means $\neg Holds(p, s)$, the same cannot be supposed for the causality predicate. For the purposes of this study, we can assume however, that $Caused$ distributes over the conjunction $\wedge$ as does $Holds$. But $Caused$ does certainly not distribute over negation, since "p is not caused to be $true$" is not the same as "p is caused to be $false$"!! We can only assume, that $Caused(\neg p, true, s)$ is to be represented by $Caused(p, false, s)$.

1. Monotonicity
   Lin's theory of causation is **monotonic**, because causal implication is just expressed by use of classical implication. It is easy to see that $Holds(f, s) \rightarrow Caused(g, v, s)$ implies $Holds(f, s) \wedge Holds(h, s) \rightarrow Caused(g, v, s)$ for any new fact (fluent) h. This means more generally, that from "f CAUSES g", we derive "$f \wedge h$ CAUSES g".

2. Transitivity
   The causality relation defined by Lin is transitive. We have to consider two cases, $Caused(g, true, s)$ and $Caused(g, false, s)$:
   $Holds(f, s) \rightarrow Caused(g, true, s)$ and
   $Holds(g, s) \rightarrow Caused(h, v, s)$, for $v \in \{true, false\}$
   implies
   $Holds(f, s) \rightarrow Caused(h, v, s)$ by use of the axioms (C) and the transitivity of classical implication.

   $Holds(f, s) \rightarrow Caused(g, false, s)$ and
   $\neg Holds(g, s) \rightarrow Caused(h, v, s)$, for $v \in \{true, false\}$
   implies
   $Holds(f, s) \rightarrow Caused(h, v, s)$ by use of the axioms (C) and the transitivity of classical implication.

3. Contraposition
   Lin's causality relation does not admit contraposition. From $Holds(f, s) \rightarrow Caused(g, v, s)$ we cannot derive $\neg Holds(g, s) \rightarrow Caused(f, v', s)$ (where $true' = false$ and $false' = true$). But the following derivation is possible in Lin's theory:

from (1)	$Holds(p, s) \rightarrow Caused(q, true, s)$	
and (2)	$Caused(q, false, s)$	
we get (3)	$\neg Holds(q, s)$	by (C) from (2)
(4)	$\neg Caused(q, true, s)$	by (C) from (3)
(5)	$\neg Holds(p, s)$	from the assumption(1) and (4)
(6)	$\neg Caused(p, true, s)$	by (C) from (5)

So, from "p causes q" and assuming that "q is caused $false$", (or that q does not hold), we can derive that "p does not hold" (5), and that p is not caused $true$ (6). But we cannot derive that p is caused $false$!

4. Reflexivity
Lin's theory is **not** reflexive. $Holds(p, s) \rightarrow Caused(p, true, s)$ is of course not an axiom of the logic and cannot be derived in Lin's theory. Instead, we have, for example

$$\begin{array}{ll} Holds(f, s) \rightarrow \neg Caused(f, false, s) & \text{by } (C) \\ \neg Holds(f, s) \rightarrow \neg Caused(f, true, s) & \\ Caused(f, false, s) \rightarrow \neg Caused(f, true, s) & \\ Caused(f, true, s) \rightarrow \neg Caused(f, false, s) & \end{array}$$

But $\neg Caused(f, v, s) \rightarrow Caused(f, v', s)$ does not hold. $Caused$ behaves like a modality operator ruled by the system T, where from $\Box p \rightarrow p$ follows $\Box p \rightarrow \neg\Box\neg p$ and $\Box\neg p \rightarrow \neg\Box p$. But $\neg\Box p \rightarrow \Box\neg p$ does **not** hold!

5. Conjunction of preconditions
Since the causal implication is formulated by use of classical implication, we can derive

$$\begin{array}{ll} \text{if} & Holds(f, s) \wedge Holds(g, s) \rightarrow Caused(h, v, s) \\ \text{then either} & Holds(f, s) \rightarrow Caused(h, v, s) \\ \text{or} & Holds(g, s) \rightarrow Caused(h, v, s) \end{array}$$

6. Conjunction
The predicate $Caused$ is not defined for complex formulas. We can deduce

$$\begin{array}{lll} \text{from} & Holds(f, s) \rightarrow Caused(g, v, s) & \text{and} \\ & Holds(f, s) \rightarrow Caused(h, v, s) & \text{follows} \\ & Holds(f, s) \rightarrow Caused(g, v, s) \wedge Caused(h, v, s) & \end{array}$$

$Caused(g \wedge h, v, s)$ is not expressible, but we can suppose that it stands for $Caused(g, v, s) \wedge Caused(h, v, s)$.

7. Reasoning by case
The "or" rule holds for Lin's causal implication, by virtue of its formulation by means of classical implication, since from

$$\begin{array}{ll} Holds(f, s) \rightarrow Caused(g, v, s) & \text{and} \\ Holds(h, s) \rightarrow Caused(g, v, s) & \text{follows} \\ Holds(f, s) \vee Holds(h, s) \rightarrow Caused(g, v, s) & \end{array}$$

8. RW
This rule does not hold. From

$$\begin{array}{ll} \models \phi \rightarrow \psi \text{ and} & \\ Holds(\chi, s) \rightarrow Caused(\phi, v, s) & \text{does not follow} \\ Holds(\chi, s) \rightarrow Caused(\psi, v, s) & \end{array}$$

We can only conclude $Holds(\chi, s) \rightarrow Holds(\psi, s)$. The reason is that while the predicate $Caused(f, v, s)$ allows to express that **fluent** (atom) f is caused to become v at state s, it does **not** give the possibility to express that **formula** ϕ is caused to become v at state s (for $v \in \{true, false\}$). Since for two different atoms p and q, $p \rightarrow q$ cannot be a logical theorem (a tautology), RW just does not apply.

9. LLE
   Since all connectives of propositional logic distribute over the *Holds*-predicate, $Holds(\phi, s)$ is equivalent to $Holds(\psi, s)$ whenever ϕ is equivalent to ψ. Consequently, from

$$\begin{array}{ll} \models \phi \leftrightarrow \psi & \text{and} \\ Holds(\phi, s) \rightarrow Caused(f, v, s) & \text{follows} \\ Holds(\psi, s) \rightarrow Caused(f, v, s) & \end{array}$$

As show examples involving exceptions, monotonicity of causal implication is a serious drawback for Lin's theory. The following exemple leads to an inconsistent theory when formulated in Lin's framework:

(1) $Holds(rain, s) \rightarrow Caused(wet, true, s)$
(2) $Holds(rain, s) \wedge Holds(has - umbrella, s) \rightarrow Caused(wet, false, s)$

Since (1) implies

$$Holds(rain, s) \wedge Holds(has - umbrella, s) \rightarrow Caused(wet, true, s)$$

we derive $holds(wet, s)$ and $\neg holds(wet, s)$ using axiom C.

3.2 McCain and Turner's causal theory for actions and change

McCain and Turner, in a series of articles [20, 21, 23] introduce a conditional type causal theory based on propositional logic and a new causal operator $\Rightarrow$. In the following we concentrate on the last published article [21]. The theory presented in [23] is shown to be equivalent to the one presented in [21]. Theorem 1 proven below can also be shown for the theory from [23].

A *causal law* is an expression of the form $\phi \Rightarrow \psi$ meaning "Necessarily, if ϕ then the fact ψ is caused". A *causal theory* is a set of causal laws.

Given a causal theory D, and an interpretation I,

$$D^I = \{\psi \ : \ I \models \phi \text{ for } \phi \Rightarrow \psi \in D\}.$$

is the set of consequents of causal laws in D whose antecedents are true in I. D^I entails exactly the formulas caused to be true in I according to D.

The central notion is the notion of causal explanation. An interpretation I is *causally explained* according to a causal theory D if I is the unique model of D^I. Of course, more than one interpretation may be causally explained by one causal theory: it can be that for two different interpretations I and J, where $I \neq J$, $[|D^I|] = \{I\}$ and $[|D^J|] = \{J\}$.

Finally, a formula ϕ is a *consequence* of a causal theory D if ϕ is true in every causally explained interpretation according to D.

It may be that a causal theory D explains no interpretation and in this case, according to the definition, every formula is a consequence of D.

Some examples might illustrate these notions.

Example 3 *Let $ATM = \{p\}$ and $D_1 = \{p \Rightarrow p\}$. The interpretation $\{p\}$ is causally explained according to D_1; since it is the only causally explained interpretation, p is a consequence of D_1.*

As the authors point out, the notions of "causal explanation" and "causal consequence" depend on the language ATM. If, in the above example, we had $ATM = \{p, q\}$, the same causal theory D_1 would explain no interpretation and would entail every formula.

The suitcase example is formulated in the following way by McCain and Turner. The language contains the non-logical symbols
$Toggle(L_1)$: the action of flipping Latch 1
$Toggle(L_2)$: the action of flipping Latch 2
$Close$: the action of closing the suitcase
$Up(L_1)$: the fluent that latch 1 is up
$Up(L_2)$: the fluent that latch 2 is up
$Open$: the fluent that the suitcase is open

The atoms of the language are expressions of the form a_t or f_t, where a and f and t are action, fluent and time names respectively.

The following are the causal laws for the suitcase example, a being a metavariable for action names (i.e. standing for $Toggle$ or $Close$) and f being a metavariable for fluent names (i.e. standing for $Up(L_i)$, $(i = 1, 2)$ or for $Open$).

Example 4

$$\begin{array}{ll}
(1) & Toggle(l)_t \wedge Up(l)_t \Rightarrow \neg Up(l)_{t+1} \\
(2) & Toggle(l)_t \wedge \neg Up(l)_t \Rightarrow Up(l)_{t+1} \\
(3) & Close_t \Rightarrow \neg Open_{t+1} \\
(4) & Up(L_1)_t \wedge Up(L_2)_t \Rightarrow Open_t \\
(5) & a_t \Rightarrow a_t \\
(6) & \neg a_t \Rightarrow \neg a_t \\
(7) & f_0 \Rightarrow f_0 \\
(8) & \neg f_0 \Rightarrow \neg f_0 \\
(9) & \sigma_t \wedge \sigma_{t+1} \Rightarrow \sigma_{t+1}
\end{array}$$

The causal laws (5) to (9) are needed for specifying that actions occur or do not occur and that facts about the initial values of fluents may be exogenous to the causal theory.

Subsequently, we show some results on causal theories which appear to be counterintuitive to what we expect from a notion of causality.

As pointed out by the authors, the consequence relation "ϕ is a *consequence* of D" is nonmonotonic, since ϕ is not necessarily a *consequence* of D' when $D \subset D'$. Adding the causal law $\neg p \Rightarrow \neg p$ to the causal theory D_1 of Example 3 yields a theory explaining the two interpretations $\{p\}$ and $\emptyset$. Thus p, which is a consequence of D_1, is not more a consequence of $D_1 \cup \{\neg p \Rightarrow \neg p\}$.

But the following theorem holds for causal theories.

Theorem 1 *Let be D a causal theory and $\phi \Rightarrow \psi \in D$. Then $\phi \to \psi$ is a consequence of D.*
Proof of et be D a causal theory and $\phi \Rightarrow \psi \in D$. We show that $\phi \to \psi$ is true in every causally explained interpretation according to D.
If no interpretation is causally explained according to D, then any formula and therefore $\phi \to \psi$ is a consequence of D.
Let be I an interpretation which is causally explained according to D. We consider two cases.
1. Case: $\psi \in D^I$. Since I is a model of D^I, I satisfies ψ and therefore I satisfies $\phi \to \psi$.
2. Case: $\psi \notin D^I$. Then, since $\phi \Rightarrow \psi \in D$, $I \not\models \phi$, hence $I \models \phi \to \psi$.
Q.E.D.

This theorem has the following immediate consequences .

Corollary 1 *Let be D a causal theory.*

1. *Contraposition*
   If $\phi \Rightarrow \psi \in D$ and $\neg\psi$ is a consequence of D then $\neg\phi$ is a consequence of D.
2. *Transitivity*
   If $\phi \Rightarrow \psi \in D$ and $\psi \Rightarrow \chi \in D$ and ϕ is a consequence of D then χ is a consequence of D.

The above theorem indicates that the consequence relation defined for a causal theory is transitive and admits contraposition. Here is the complete discussion of all the properties.

1. Monotonicity
   McCain and Turner's consequence relation is not monotonic. As illustrated by Example 3, adding a new causal law can revise the consequences of a causal theory.
2. Transitivity
   As shows Corollary 1, 2, McCain and Turner's consequence relation is transitive, since for the theory D, containing $\phi \Rightarrow \psi$ and $\psi \Rightarrow \chi$, χ is a consequence of D, whenever ϕ is.
3. Contraposition
   McCain and Turner's consequence relation admits contraposition, since $\neg\psi$ is a consequence of D, whenever D contains $\phi \Rightarrow \psi$ and $\neg\phi$ is a consequence of D.
4. Reflexivity
   Since the consequence relation is defined between two different sets, a set of causal laws and a set of formulas, i. e. **formula** ϕ is a consequence of a set of **causal laws** $\phi \Rightarrow \psi$, reflexivity cannot be considered for the consequence relation.
5. Conjunction of preconditions
   Analogous remark as for reflexivity. The preconditions of the consequence relation are causal laws, for which conjunction is not defined.
6. Conjunction
   McCain and Turner's consequence relation admits conjunction, since $\phi \wedge \psi$ is a consequence of D, whenever ϕ and ψ is. Interpretation I satisfies $\phi \wedge \psi$ whenever it saisfies ϕ and ψ.
7. Reasoning by case
   As for reflexivity and conjunction, this property cannot be considered, the left hand side of the consequence relation not being a formula.
8. RW
   Right weakening holds for McCain and Turner's consequence relation. As for conjunction, interpretation I satisfies ψ whenever it satisfies ϕ and $\models \phi \rightarrow \psi$.
9. LLE
   As for reflexivity, conjunction of preconditions and reasoning by case, this property cannot be considered for equivalent reasons.

McCain and Turner's theory defines a consequence relation between causal theories and formulas. But the causal laws themselves express also a causality relation between formulas. As stated by the authors, $\phi \Rightarrow \psi$ means "Necessarily, if ϕ then the fact ψ is caused". On the other hand, causal rules behave much like meta rules the causality connector $\Rightarrow$ has no specific syntactical properties. However, we have considered the behaviour of causal theories with respect to the consequence relation when we add new causal rules to a causal theory D, which are "derived" from rules in D.

Let us first show these results.

Theorem 2 *Let D be a causal theory and $\phi \Rightarrow \psi \in D$. Then D has the same consequences as $D \cup \{\phi \wedge \chi \Rightarrow \psi\}$.*

In the same sense, $\Rightarrow$ is transitive.

Theorem 3 *Let D be a causal theory and $\phi \Rightarrow \psi$, $\psi \Rightarrow \rho \in D$. Then D has the same consequences as $D \cup \{\phi \Rightarrow \rho\}$.*

Theorem 4 *Let D be a causal theory and $\phi \Rightarrow \psi \in D$ and $\phi \Rightarrow \chi \in D$. Then $D \cup \{\phi \Rightarrow \psi \wedge \chi\}$ has the same consequences as D.*

Theorem 5 *Let D be a causal theory and $\phi \Rightarrow \psi \in D$ and $\chi \Rightarrow \psi \in D$. Then $D \cup \{\phi \vee \chi \Rightarrow \psi\}$ has the same consequences as D.*

Theorem 6 *Let D be a causal theory and $\phi \Rightarrow \psi \in D$ and $\models \psi \rightarrow \chi$. Then $D \cup \{\phi \Rightarrow \chi\}$ has the same consequences as D.*

The proof of the theorems can be found in the appendix.

Theorems 2 and 3 suggest that the causality relation $\Rightarrow$ is nonmonotonic and transitive. Let us consider the complete list of properties we discussed for causal implication for the causality relation $\Rightarrow$.

1. Monotonicity
   McCain and Turner's causal implication $\Rightarrow$ is monotonic, since the theory D containing $\phi \Rightarrow \psi$ has the same consequences as the theory $D' = D \cup \{\phi \wedge \chi \Rightarrow \psi\}$ according to Theorem 2.

2. Transitivity
   According to Theorem 2, McCain and Turner's causal implication $\Rightarrow$ is transitive, since the theory D containing $\phi \Rightarrow \psi$ and $\psi \Rightarrow \chi$ has the same consequences as the thery $D' = D \cup \{\phi \Rightarrow \chi\}$.

3. Contraposition
   McCain and Turner's causal implication $\Rightarrow$ does not admit contraposition, since for $\phi \Rightarrow \psi \in D$, $D \cup \{\neg\psi \Rightarrow \neg\phi\}$ has not the same consequences as the theory D.

4. Reflexivity
   McCain and Turner's theory is **not** reflexive, since the causal theory D augmented with reflexive laws $\phi \Rightarrow \phi$ does not necessarily have the same consequences as D, as shows the Example 3.

   However, in applications to action theories, the authors use reflexive laws $a \Rightarrow a$ and $\neg a \Rightarrow \neg a$ for all action names a as well as $f_0 \Rightarrow f_0$ and $\neg f_0 \Rightarrow \neg f_0$ for all fluent names f_0. This is necessary in order to express that actions might (or might not) occur and that fluents "exist" at some first state of a world.

5. Conjunction of preconditions
   We can have that $\phi \wedge \chi \Rightarrow \psi \in D$ and neither $D \cup \{\phi \Rightarrow \psi\}$ nor $D \cup \{\chi \Rightarrow \psi\}$ have the same consequences as D.

6. Conjunction
McCain and Turner's theory admits conjunction, since for $\phi \Rightarrow \psi \in D$ and $\phi \Rightarrow \chi \in D$, $D \cup \{\phi \Rightarrow \psi \wedge \chi\}$ has the same consequences as D according to Theorem 4.
7. Reasoning by case
McCain and Turner's theory admits reasoning by case. If $\phi \Rightarrow \psi \in D$ and $\chi \Rightarrow \psi \in D$, then D has the same consequences as $D \cup \{\phi \vee \chi \Rightarrow \psi\}$ according to the Theorem 5.
8. RW
Theorem 6 shows that McCain and Turner's theory admits right weakening.
9. LLE
Since causal explanation and consequence of a causal theory are defined using satisfiability of a formua by a model, the rule RCK is valid for the consequence relation. We mean by that, that a causal theory D has the same consequences as $D \cup \{\phi' \Rightarrow \psi\}$ whenever D contains $\phi \Rightarrow \psi$ and $[|\phi|] = [|\phi'|]$.

3.3 Thielscher's Theory of Ramification and Causation

Thielscher [22] presents a STRIPS-like approach augmented by causal rules which are automatically derived from domain constraints by means of "influence information". Based on a propositional language, an action is defined as a triplet (C, a, E), where C is the set of precondition literals, E is the set of result literals and a is the action name, and $\mid C \mid = \mid E \mid$ [1], i.e., the preconditions and the effects refer to the same set of fluent names. This restriction seems heavy. For an example, the action of moving an object a from place p to place q cannot be formulated by $(\{atap)\}, movea, \{ataq)\})$, but has to become $(\{atap, \neg ataq\}, movea, \{ataq, \neg atap\})$, even in the presence of the constraint $\neg(atap \wedge ataq)$.

Constraints formulated in classical logic bby means of material implication. In order to take into account causality, Thielscher proposes a method for automatically generating a set of *causal* rules from a given set of classical domain constraints, which exploits the information of potential influence of some fluents upon others. What he calls "influence information" is a binary relation I between fluents, where $(f_1, f_2) \in I$ means that f_1 might possibly influence f_2. Given a constraint in clausal form $l_1 \vee \cdots \vee l_n$ and the influence information $(\mid l_i \mid, \mid l_j \mid) \in I$, he generates automatically the causal law

$$\neg l_i \ causes \ l_j \ if \bigwedge_{1 \leq k \leq n, k \neq i, k \neq j} \neg l_k$$

The original contribution of Thielscher's work is this capacity to derive causal rules automatically by means of influence information. But it is probably not easy to establish such an influence relationship. Thielscher doesn't indicate how influence information is established. He gives the relation I by explicit enumeration supported by the intuitive idea "if $(f_1, f_2) \in I$ then this is intended to denote that a change of f_1's truth value might possibly influence the truth value of f_2". A notion rather similar to that of "influence"

[1]Given a literal l, $\mid l \mid$ denotes its atomic subformula, i.e. $\mid l \mid = l$ if l is a positive literal, and $\mid l \mid = \neg l$ if l is a negative literal. For a set of literals M, $\mid M \mid = \{\mid l \mid : l \in M\}$

was studied by Fariñas and Herzig in [2], where they define an "interference" relation in a very similar way. The work is also related to some work by Dubois et al. [1] who go further and study axiomatics and semantics for dependency relations. Gärdenfors in [4] presents axiomatic approaches to relevance and proposes semantics for relevance within a probabilistic framework. Furthermore, the relationships between dependency and update (and theory revision), which are very important for the use of causality rules within action theories, have been studied, e.g. by Gärdenfors in [5] and by Fariñas and Herzig in [3].

The effects of the execution of an action (C, a, E) in a state S (states are always complete in Thielscher's framework) are computed by replacing C within S by E (provided that the action is possible, i.e. $C \subset S$) yielding an intermediate result state S' and then by applying causal rules, i.e. by successively adding literals l' to S' for every $l \in S'$ whenever l *causes* l' if γ is a causal law and $\gamma \subset S'$ and suppressing $\neg l' \in S'$. Thielscher's system solves the frame problem in a simple and efficient way, since all what is not changed by an action or by a causal law remains in the state and thus persists.

The suitcase example is formulated as follows in Thielscher's theory.

Example 5

$$
\begin{array}{ll}
\textit{Action laws} & (up1, flip1, \neg up1) \\
 & (\neg up1, flip1, up1) \\
 & (up2, flip2, \neg up2) \\
 & (\neg up2, flip1, up2) \\
\textit{Constraint} & \neg up1 \vee \neg up2 \vee open \\
\textit{Influence} & I = \{(up1, open), (up2, open)\}
\end{array}
$$

¿From the constraint and the influence information, the following set of causal rules is generated:

$$
\begin{array}{l}
up1\ causes\ open\ if\ up2 \\
up2\ causes\ open\ if\ up1
\end{array}
$$

Given an initial state $\{\neg up1, up2, \neg open\}$ *the action* $flip1$ *generates an intermediate state* $S' = \{up1, up2, \neg open\}$ *and application of the causal law "*$up1\ causes\ open\ if\ up2$*" has the effect to replace* $\neg open$ *in* S' *by* $open$ *yielding the final state* $\{up1, up2, open\}$.

We now study the behaviour of Thielscher's theory with respect to the properties we exhibited in the last chapter. We first want to point out that the language of Thielscher's theory heavily restricts what is expressible concerning causality. Influence information is defined for pairs of atoms p and q, (p influences q). Causal laws are of the form $l\ causes\ l'\ if\ \bigwedge l_i$, where l, l' and all l_i are literals. Consequently, some of our properties for causal implication do not apply in Thielscher's causality relation merely because they cannot be expressed in his language.

1. Monotonicity
   This property cannot be expressed, since there is no causal law of the form "$\phi \wedge l\ causes\ l'\ if\ \ldots$". However, constraints are clauses and the set of clauses $\{c_1, c_2, \ldots, c_n\}$ is equivalent the set $\{,_1, ,_2, \ldots, ,_{c_i} \vee l\}$, where l is any literal. Taking into account these new clauses, which do not alter the meaning of the set of constraints, Thielscher system would produce a new causal law "$\neg l_i\ causes\ l_j\ if\ \bigwedge l_i \wedge l$", whenever $(| l_i |, | l_j |) \in I$. This is entirely correct.

2. Transitivity
   Thielscher's causal implication is not transitive, since the influence relation on which depends causal implication is not transitive. Hence from l *causes* l' *if* ... and l' *causes* l'' *if* ... we cannot conclude l *causes* l'' *if* ... unless $(l, l'') \in I$, which does not follow from $(l, l') \in I$ and $(l', l'') \in I$.

3. Contraposition
   Thielscher's causal implication does not admit contraposition, unless the influence relation I is symmetric, which is not the case.

4. Reflexivity
   Thielscher's causal implication is not reflexive, since I is not reflexive. Unless $(l, l) \in I$, "l *causes* l *if* ..." cannot be generated.

5. Conjunction of preconditions
   This property cannot be expressed, since conjunctions do not occur in influence information and consequently cannot occur as preconditions in causal implications. The bain tub example cannot be expressed in Thielscher's theory.

6. Conjunction
   This property cannot be expressed, since conjunctions are not admitted as consequences of a causal rule.

7. Reasoning by case
   This property cannot be expressed, since disjunction cannot occur as precondition of a causal rule.

8. RW
   Again, this property cannot be expressed in Thielscher's theory.

9. LLE
   As for conjunction and disjunction, this property cannot be expressed in Thielscher's theory. Two different literals cannot be logically equivalent.

Thielscher's theory is based on a representation of the constraints in classical logic. Therefore, laws with exceptions cannot be represented without getting into contradictions with actual situations. For an example, consider the suitcase example augmented by one law, expressing that the suitcase may be closed by a leather belt (the literal *belt* represents the fact that there is a leather belt arround the suitcase and closing it).

Example 6

$$
\begin{array}{ll}
\textit{Action laws} & (up1, flip1, \neg up1) \\
 & (\neg up1, flip1, up1) \\
 & (up2, flip2, \neg up2) \\
 & (\neg up2, flip1, up2) \\
\textit{Constraints} & \neg up1 \vee \neg up2 \vee open \\
 & \neg belt \vee \neg open \\
\textit{Influence} & I = \{(up1, open), (up2, open), (belt, \neg open)\}
\end{array}
$$

¿From the constraint and the influence information, the following set of causal rules is generated:

$$
\begin{array}{l}
up1 \ \textit{causes} \ open \ \textit{if} \ up2 \\
up2 \ \textit{causes} \ open \ \textit{if} \ up1 \\
belt \ \textit{causes} \ \neg open
\end{array}
$$

As an immeadiate consequence of the constraints, we have $\neg(up1 \wedge up2 \wedge belt)$*, contradicting the state* $(up1 \wedge up2 \wedge belt)$*. However, this state should be possible in an action theory (the suitcase might be closed by a leather belt with both latches up).*

3.4 Giordano, Martelli and Schwind's dynamic causal action logic

L. Giordano, A. Martelli and C. Schwind in [7, 8] have defined a dynamic logic for actions and causality. Actions are represented by modalities. If a is an action and ϕ is a formula, then $[a]$ is a modal operator and the formula $[a]\phi$ means that ϕ is true after the execution of a. Causality is expressed by a unary modal operator © and ©ϕ means that ϕ is caused. Both, the action operators and the causal operator are ruled by the axioms of the modal logic K. This dynamic language for actions is used for formulating action systems. An action system is characterized by two sets of laws, one is a domain description and the other is a set of persistency laws. The domain description contains action laws, causality laws and constraints. All laws in a domain description are true in all situations of an action world. The persistency laws are laws of the form $l \rightarrow [a]l$, where l is a literal and a is an action name. The set of persistency laws for a theory is defined by a relation $Frame$ between actions and positive literals. In most action theories, $Frame$ is just $ACT \times ATM$, where ACT is the set of all action names and ATM is the set of atoms of the language. They introduce a notion of extension (of a domain description in a set of persistency laws), where an extension is obtained by adding a maximal set of persistency laws to a domain description.

More formaly, an extension is defined as follows.

Definition 1 *Let* $D = (\Pi, Frame)$ *be a domain frame. An* extension *of* D *is a set* $E = Th_{AD}(\Pi \cup WP' \cup F)$*, such that* $WP' \subseteq WP_{Frame}$ *and* $F \subseteq Ass_{NonFrame}$ *and*

(a) *if* $(|l|, a) \in Frame$*, then* $[A](l \rightarrow [a]l) \in WP' \iff [A; a]\neg l \notin E$

(b) *if* $[A]l \in Ass_{NonFrame}$ *then* $[A]l \in F \iff [A]\neg l \notin E$

Here is the suitcase example formulated in Giordano's Martelli's and Schwind's action theory.

Example 7 Π *contains action and causality laws, Obs describes the initial state (preceded by the "empty" action sequence* $[\varepsilon]$ *and every literal is a* $Frame$ *literal, i.e. may indirectly change.*

$$
\begin{array}{lll}
\Pi: & \Box(\neg up_1 \rightarrow [flip_1]up_1) & \Box(up_1 \rightarrow [flip_1]\neg up_1) \\
 & \Box(\neg up_2 \rightarrow [flip_2]up_2) & \Box(up_2 \rightarrow [flip_2]\neg up_2) \\
 & \Box(up_1 \wedge up_2 \rightarrow \copyright open) & \\
Obs: & [\varepsilon](\neg up_1 \wedge \neg up_2 \wedge \neg open) & \\
\multicolumn{3}{l}{Frame = \{(l, a) : a \in \mathcal{A}, l \in \mathcal{F}\}.}
\end{array}
$$

The first four rules in Π describe the immediate effects of the action of flipping the status of a lock, whereas the fifth rule is a causal rule which describes the effect of the spring loaded mechanism. All the fluents are considered as being persistent. If we have an initial situation, where up_1 is not *up* and the suitcase is not *open* and up_2 is *up*, $\neg up_1 \wedge up_2 \wedge \neg open$, and the action $[flip_1]$ occurs as a result we have $[flip_1]up_1$. This theory has one extension where $[flip_1]open$ holds. There is no extension where $\neg open$ persists and $[flip_1]\neg up_2$ holds because the causal law "$up_1 \wedge up_2 \rightarrow ©open$" is not equivalent to "$up_1 \wedge \neg open \rightarrow ©\neg up_2$".

We now study the behaviour of Giordano, Martelli and Schwind's theory with respect to the properties we exhibited in the last chapter. We want to point out that, as for Lin's theory, causality here is defined by means of classical implication. Causal laws are of the form $\phi \rightarrow ©\psi$, where ϕ and ψ are formulas. Consequently, some of the properties of classical implication will also hold for causal implication.

1. Monotonicity
   Since the causality relation in dynamic action logic is expressed by means of classical implcation, it is monotonic. From $\alpha \rightarrow ©\beta$ we can of course conclude $\alpha \wedge \gamma \rightarrow ©\beta$.
2. Transitivity
   Giordano, Martelli and Schwind's causal implication is not transitive. From $\alpha \rightarrow ©\beta$ and $\beta \rightarrow ©\gamma$, we cannot conclude $\alpha \rightarrow ©\gamma$.
3. Contraposition
   Giordano, Martelli and Schwind's causal implication does not admit contraposition. Of course, as illustrated by the Example 7, from $\alpha \rightarrow ©\beta$ follows, $\neg©\beta \rightarrow \neg\alpha$, but *not* $\neg\beta \rightarrow ©\neg\alpha$.
4. Reflexivity
   Giordano, Martelli and Schwind's causal implication is not reflexive. $\alpha \rightarrow ©\alpha$ is not a theorem of the dynamic action logic.
5. Conjunction of preconditions
   This property holds for Giordano, Martelli and Schwind's causal implication. ¿From $\alpha \wedge \beta \rightarrow ©\gamma$, we conclude $\alpha \rightarrow ©\gamma$ or $\beta \rightarrow ©\gamma$. This is due to the formulation of causality by means of classical implication.
6. Conjunction
   This property holds, because © distributes over $\wedge$. This is a general property of all modal operators of type K. We have $©(\alpha \wedge \beta) \leftrightarrow ©\alpha \wedge ©\beta$. And therefore, from $\gamma \rightarrow ©\alpha$ and $\gamma \rightarrow ©\beta$, we can conclude $\gamma \rightarrow ©(\alpha \wedge \beta)$.
7. Reasoning by case
   Giordano, Martelli and Schwind's causal implication admits reasoning by case. ¿From "$\alpha \rightarrow ©\gamma$" and "$\beta \rightarrow ©\gamma$" we conclude "$\alpha \vee \beta \rightarrow ©\gamma$" due to the property of classical implication.
8. RW
   RCK holds for Giordano, Martelli and Schwind's causal implication. By means of the axiom (K), which is $©(\alpha \rightarrow \beta) \rightarrow ©\alpha \rightarrow ©\beta$, we can conclude $\alpha \rightarrow ©\gamma$ from $\alpha \rightarrow ©\beta$ and $\vdash \beta \rightarrow \gamma$.
9. LLE
   Again, this property holds for Giordano, Martelli and Schwind's causal implication, because a equivalent subformulas may be replaced by each other.

The example from 1.1 cannot be correctly represented in this theory. Due to monotonicity of causal implication it would yield an inconsistent theory.

3.5 Summary tableau

The following table summarizes the results of this section. We expect that the five first properties do not hold for causal implication whereas the four last properties should hold. But as has been shown, this is not always the case.

Criteria	Lin	MT cons	MT $\Rightarrow$	Thiel	GMS
Monotonicity	yes	no	yes	-	yes
Transitivity	yes	yes	yes	no	no
Contraposition	no	yes	no	no	no
Reflexivity	no	-	no	no	no
Conj. of prec.	yes	-	no	-	yes
AND	yes	yes	yes	-	yes
OR	yes	-	yes	-	yes
RW	-	yes	yes	-	yes
LLE	yes	-	yes	-	yes

MT cons stands for McCain and Turner's consequence relation, Thiel for Thielscher's and GMS for Giordano, Martelli and Schwind. We filled in "yes" whenever the respective theory admits the property and "no" if not. In the cases where the property is not expressible, we put -. It is worth observing that we did not meet any ideal theory admitting all the properties we thought it should and not admitting non one of those it should not.

4 Conclusion

In this article, we made an attempt to elaborate general principles for defining and using causality in action theories. We discussed several approaches to actions and causality along these principles and gave a comparative tableaux.

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Appendix: Proofs of the theorems

Let us first observe the following:

Lemma 1 *Let D, D' be causal theories, α any propositional formula and I any propositional interpretation. Then*

1. *If $D \subset D'$ then $D^I \subset D'^I$*
2. *If $I \not\models \alpha$ then for any formula χ, $D^I = (D \cup \{\alpha \Rightarrow \chi\})^I$.*
3. *If $\psi \in D^I$ then for any formula ϕ, $D^I = (D \cup \{\phi \Rightarrow \psi\})^I$.*

The proof is trivial.

Proof of Theorem 2

Let be D a causal theory and $\phi \Rightarrow \psi \in D$. Then D has the same consequences as $D' = D \cup \{\phi \wedge \chi \Rightarrow \psi\}$.
Proof of e first note that $D^I = D'^I$, since the law added to D has the same consequence as one law from D and the latter has a stronger antecedent. Hence D^I and D'^I have the same models, that is the only model I. Consequently, I is causally explained by D if and only if I is causally explained by D'. Therefore, D and D' have the same consequences. Q.E.D.

Proof of Theorem 3

Let be D a causal theory and $\phi \Rightarrow \psi$, $\psi \Rightarrow \rho \in D$. Then D has the same consequences as $D' = D \cup \{\phi \Rightarrow \rho\}$.
Proof of e show that $D^I = D'^I$, for any interpretation I causally explained by D. Let be $\rho \in D'^I$, by application of the new rule $\phi \Rightarrow \rho$. Then $I \models \phi$ and therefore $\psi \in D^I$. Since I is a model of D^I, $I \models \psi$ and therefore $\rho \in D^I$. Hence, $[|D'^I|] = \{I\}$, i.e. I is also causally explained by D'. On the other hand, we show that $D'^J = D^J$, for any interpretation J causally explained by D'. Let be $\rho \in D'^J$ by application of the new rule $\phi \Rightarrow \rho$. Then $J \models \phi$ from which follows that $\psi \in D^J$ and since $[|D'^J|] = \{J\}$, $J \models \rho$. Now suppose that $\rho \notin D^J$. then $J \not\models \psi$ (since $\psi \Rightarrow \rho \in D$), from which follows that $\psi \notin D'^J$. But this entails that $J \not\models \phi$ which is a contradiction! Hence the same interpretations are explained causally by D and D', from which follows immediately the assertion of the theorem.
Q.E.D.

Proof of Theorem 4

Let D be a causal theory and $\phi \Rightarrow \psi \in D$ and $\phi \Rightarrow \chi \in D$. Then $D' = D \cup \{\phi \Rightarrow \psi \wedge \chi\}$ has the same consequences as D.
Proof of e show that $[|D^I|] = [|D'^I|]$ for any interpretation I, which then entails that D and $D \cup \{\phi \Rightarrow \psi \wedge \chi\}$ have the same consequences. It is easy to see that $D'^I = D^I \cup \{\psi \wedge \chi : \phi \Rightarrow \psi \in D \text{ and } \phi \Rightarrow \chi \in D \text{ and } I \models \phi\}$. Obviously, the two sets D^I and D'^I have the same models, since $I \models \psi \wedge \chi$ if and only if $I \models \psi$ and $I \models \chi$.Q.E.D.

Proof of Theorem 5

Let D be a causal theory and $\phi \Rightarrow \psi \in D$ and $\chi \Rightarrow \psi \in D$. Then $D' = D \cup \{\phi \vee \chi \Rightarrow \psi\}$ has the same consequences as D.
Proof of e show that $D^I = D'^I$ for any interpretation I. Let be $\psi \in D'^I$ by application of the new rule $\phi \vee \chi \Rightarrow \psi$. Then $I \models \phi \vee \chi$ from which follows that $I \models \phi$ or $I \models \chi$. Either of the cases implies that $\psi \in D^I$

Q.E.D.

Proof of Theorem 6

Let D be a causal theory and $\phi \Rightarrow \psi \in D$ and $\models \psi \rightarrow \chi$. Then $D' = D \cup \{\phi \Rightarrow \chi\}$ has the same consequences as D. Proof of et be I causally explained by D. We show first that $[|D^I|] = [|D'^I|]$. Since $D^I \subset D'^I$, $[|D'^I|] \subset [|D^I|]$. Since $[|D^I|] = \{I\}$, we have to show that $I \in [|D'^I|]$. Suppose for the contrary that this is not the case. Then there is $\chi \in D'^I$ such that $I \not\models \chi$ and χ is the consequent of the new rule $\phi \Rightarrow \chi$ added by right weakening to D; i.e. $\phi \Rightarrow \psi \in D$ and $\models \psi \rightarrow \chi$ and $I \models \phi$. Since $I \not\models \chi$, $I \not\models \psi$. Since $I \in [|D^I|]$, $\psi \notin D^I$, from which follows that $I \not\models \phi$, which gives a contradiction.

Q.E.D.