Matematisk modellering och problemlösning

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Computer Science
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Linköping, March 27, 2013

Course home page at http://www.cse.chalmers.se/edu/year/2010/course/DAT026/
or via my personal home page.

An inquiry-based course in mathematical modelling and problem solving

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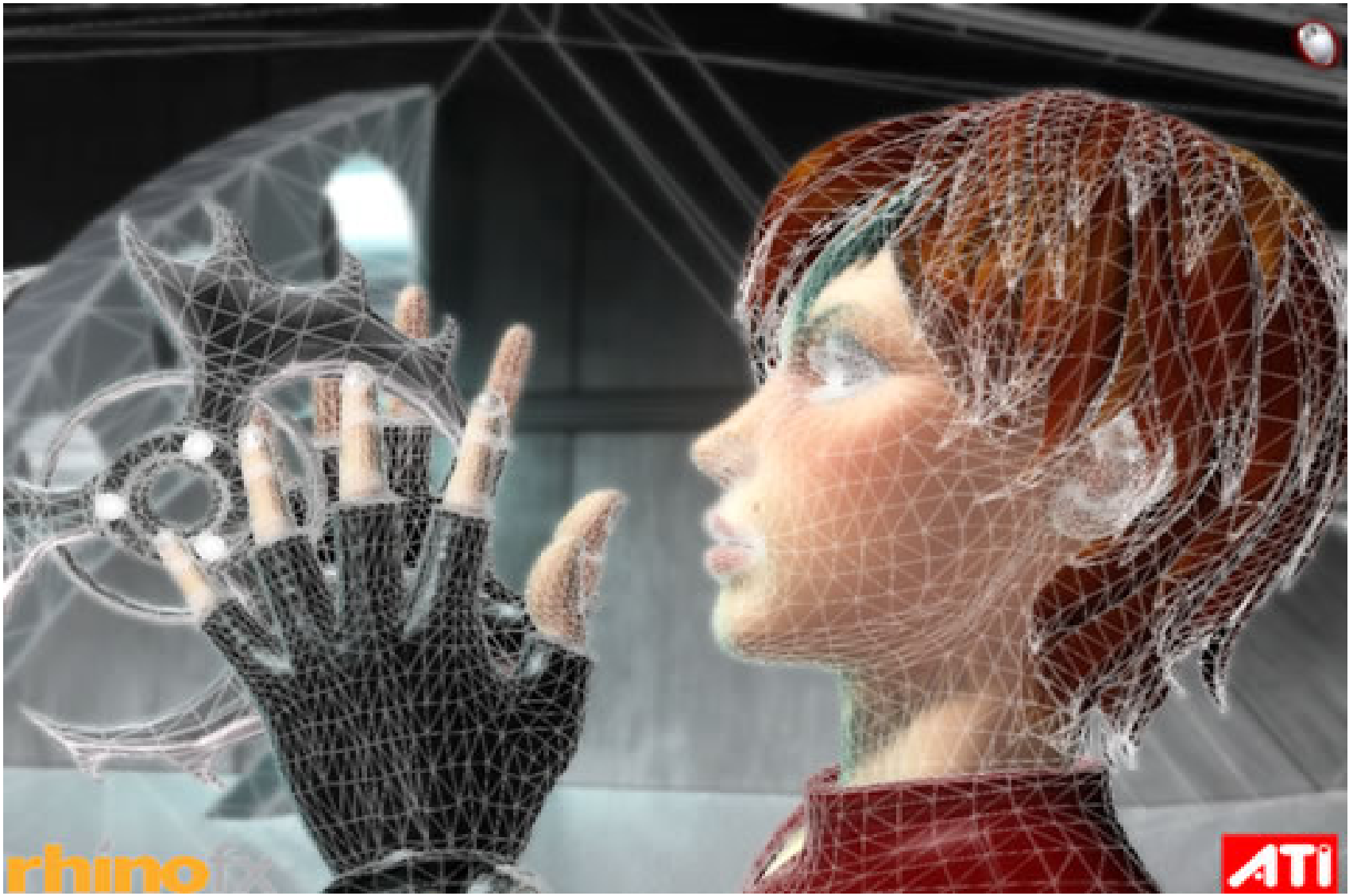
My starting point

Many software engineering and and computer science students do not understand where and how to use mathematics.

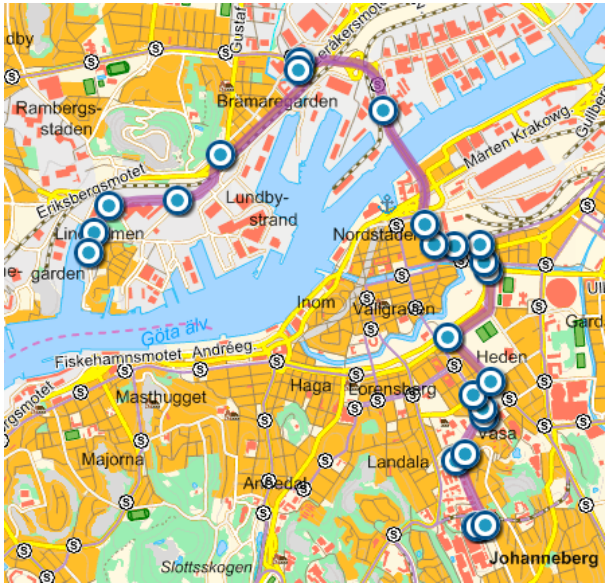
Not even the mathematics they already know!

```
import java.awt.*;

public abstract class Animation
  extends java.applet.Applet
    implements java.lang.Runnable {

    protected Dimension d;  // bitmap size
    protected Image im;  // extra image for drawing
    protected Graphics offscreen; // the offscreen bitmap to draw in
    protected int delay = 100; // in milliseconds
    protected Thread animationThread;

    final public void init() {
   d = getSize();
   im = createImage(d.width, d.height);
   offscreen = im.getGraphics();
   initAnimator();
    }

    //final public void paint(Graphics g) {update(g);}

    final public void update(Graphics g) {
   paintAnimator(offscreen);  // first draw offscreen to reduce flicker
   g.drawImage(im, 0, 0, this);  // then put on screen
    }

    // To be implemented in subclass that does the actual drawing
    protected void initAnimator() {} // init for drawing routines
    abstract protected void paintAnimator(Graphics g); // the actual
drawing will be here

    public void setDelay(int d) {delay=d;}

    public void start() {
   animationThread = new Thread(this);
   animationThread.start();
    }

    public void stop() {
```

The math is often hidden!

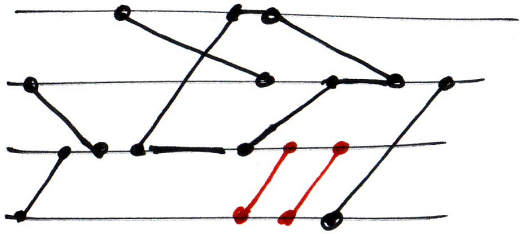


Show how different kinds of mathematics is used in different situations



$$\text{minimize} \sum_{ij \in A} w_{ij} x_{ij} \qquad \text{subject to} \qquad x \geq 0$$

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1, & \text{if } i = s; \\ -1, & \text{if } i = t; \\ 0, & \text{otherwise.} \end{cases}$$



A gap between theory and practice also in problem solving

What we tell students when we teach problem solving

What scientists and engineers actually do when they solve problems

We must consider the whole problem solving process!

Target two missing but essential generic skills

Mathematical modelling: the ability to translate real-world problems into mathematical problems

Structured problem solving: the ability to systematically explore and solve new and complex problems

Very little new mathematics

What kind of mathematics is relevant?

Difficult to know in what application areas students will work!

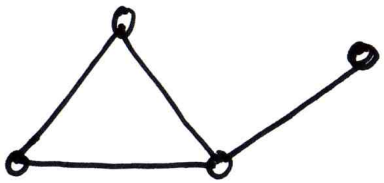
Both continuous and discrete mathematics

Several variables

Mathematical software tools

A broad perspective on models

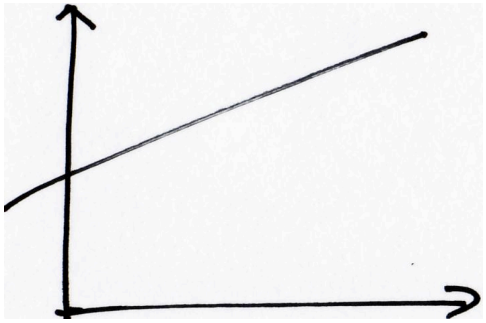
$$F = G \frac{m_1 m_2}{r^2}$$

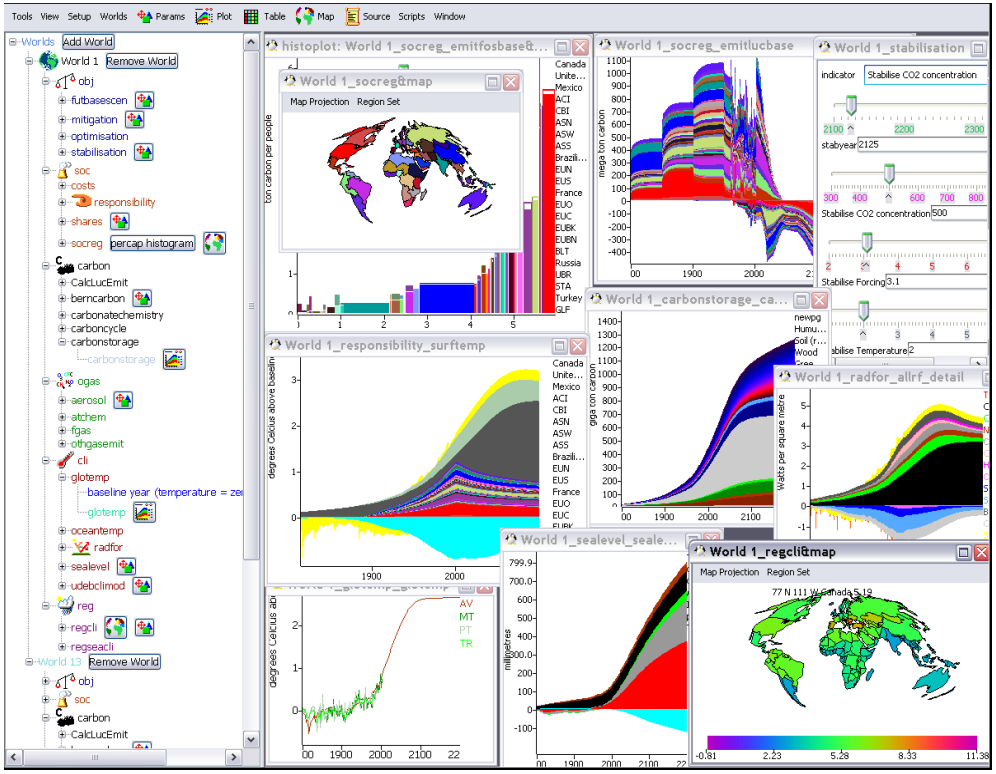


man(Socrates)

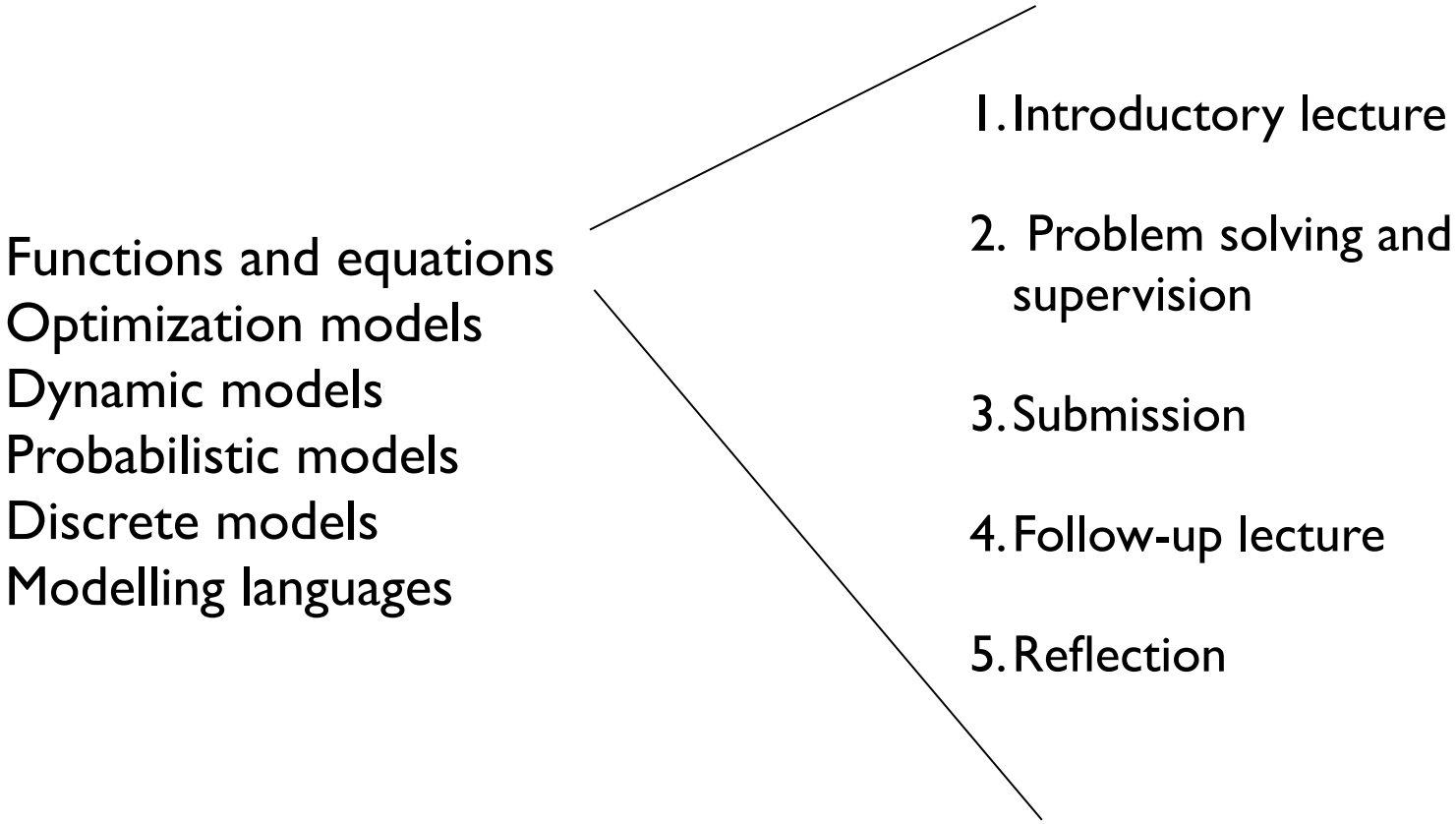
man(X) $\Rightarrow$ mortal(X)

Simple and complicated models





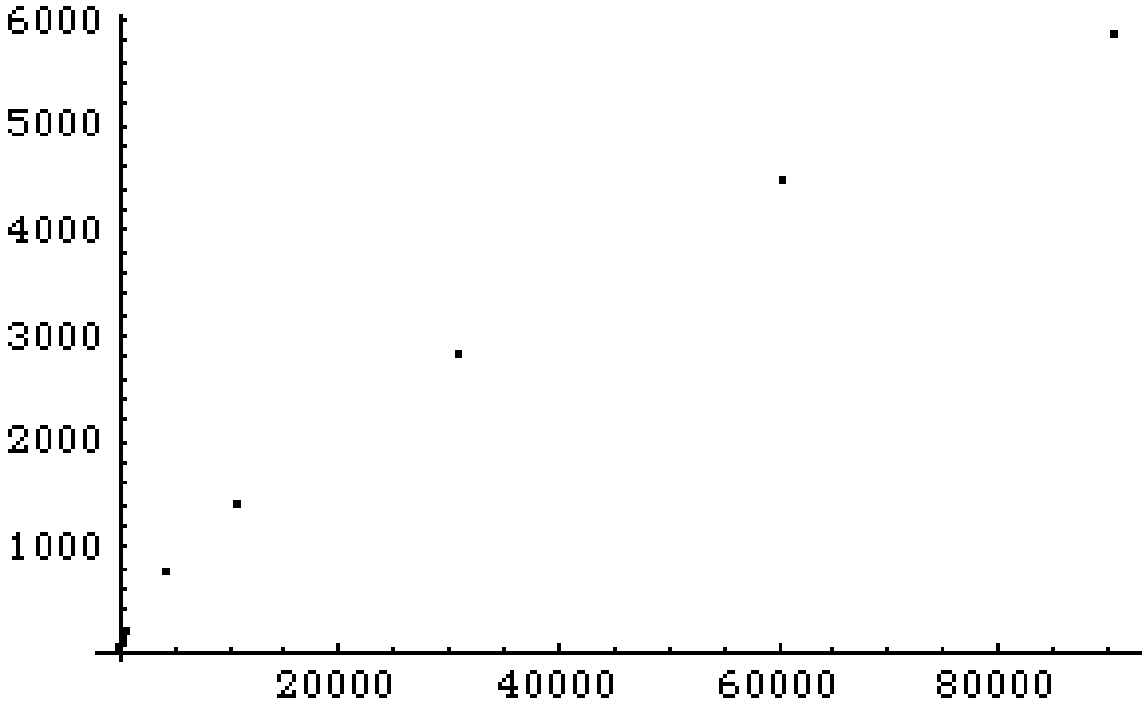
Weekly modules based on model type



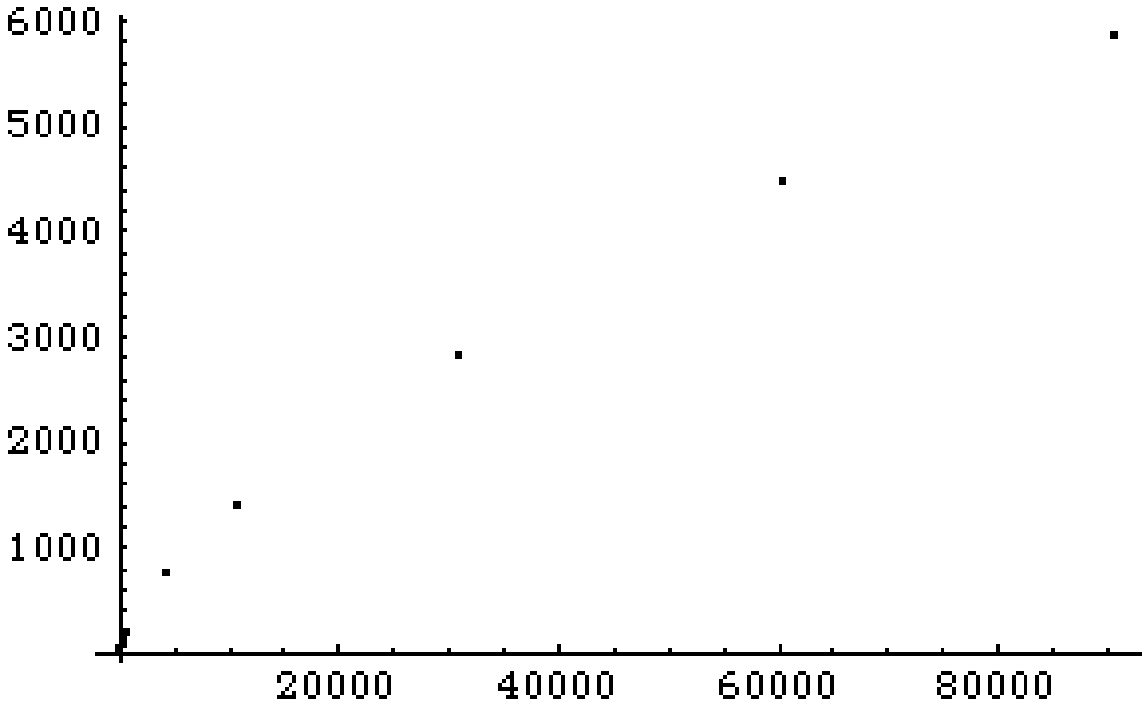
Inquiry-based learning example

T (time)	D (distance)
88.0	57.9
224.7	108.2
365.3	149.6
687.0	228.07
4332	778.434
10760	1428.74
30684	2839.08
60188	4490.8
90467	5879.13

Inquiry-based learning example



Inquiry-based learning example



Solving the problem becomes a form of exploration!

How teach exploration?

Challenging problems

Interact with students

Supervise by asking questions

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Challenging problems

Interact with students

Supervise by asking questions

Teach problem solving strategies

Adapt assessment

1. Understand the problem!
2. Plan the steps of your problem solving! Don't rush to the solution!
3. Be careful when you carry out your plan!
4. Check your results and conclusions! Reflect over the solution process!

Always do something! Never stop! You can always try to understand the problem better. Draw a figure. Solve a simpler problem. …

(1-4 freely adapted from Polya)

How teach exploration?

Challenging problems

Interact with students

Supervise by asking questions

Teach problem solving strategies

Adapt assessment

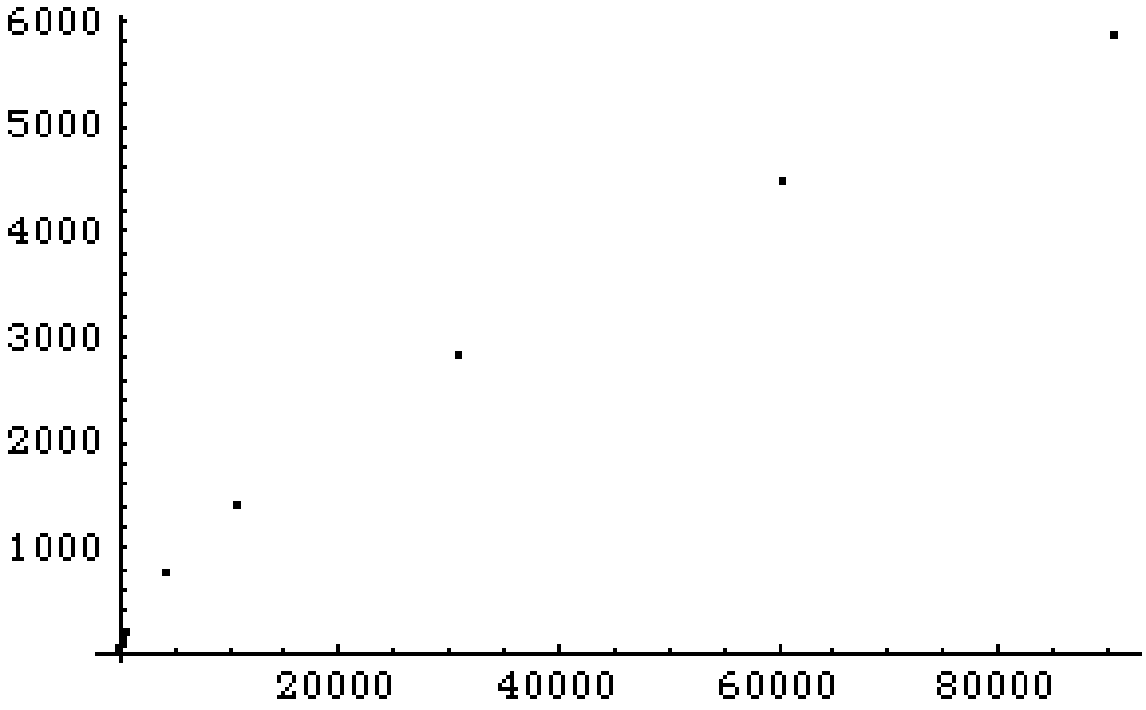
Focus on motivation and encouragement. Formal assessment in background.

Errors are allowed and natural. Accept all serious attempts even if they are wrong or incomplete.

Give continuous feedback individually and with follow-up lectures.

Final report at end of course.

Defining the course with problems



- introduce curve fitting
- develop your own problem solving strategy
- recapitulate basic mathematical functions
- evaluate quality of model
- prepare for the least squares method in a later exercise
- link to the history of science

More examples of problems

Facility location

Lunch problem

Bouncing balls

Medicine dose

Funny text (and music)

Predict weather

Project planning

Expert system

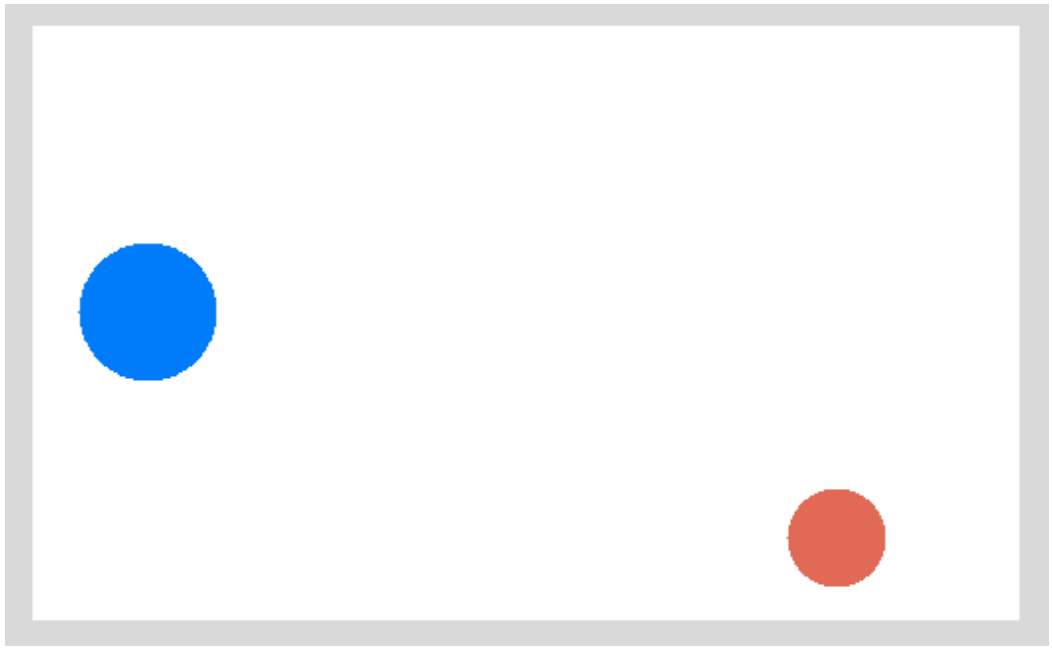
...

In total 34 problems plus reflection.

Some theoretical, for some Mathematica or other system is useful.

One larger programming exercise.

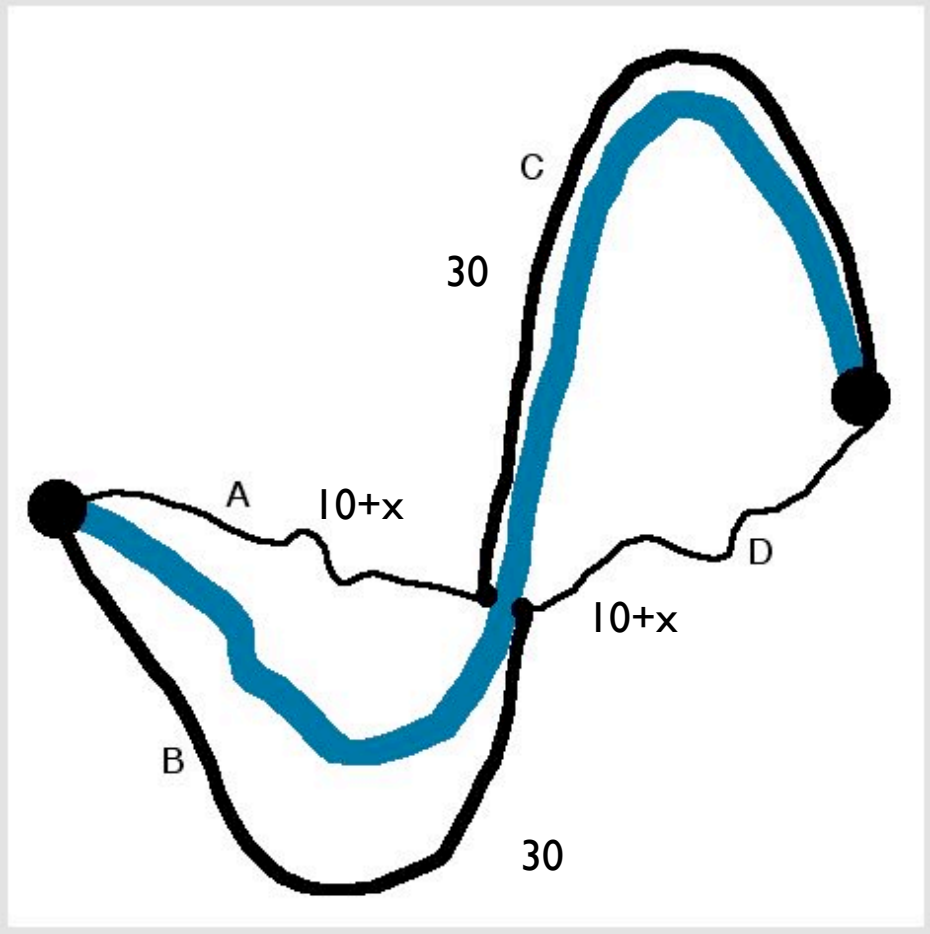
Bouncing balls simulation



Gravity

Elastic shock

Bridge problem



20 cars per minute

a) Travel time without bridge?

b) Travel time with bridge?

Our version of inquiry-based learning

Learning is *inititated* by (realistic) problems - same as for PBL, projects etc.

Many smaller problems allow *regular feedback* and *variation*

Detailed course objectives can be met

Problem design criteria: *reasonably realistic, motivating, challenging, multipurpose*

Work in pairs enables a dialogue but still ensures that every student is fully involved

How do students change by the course?

Prior limitations in students' relation to mathematics

Low self confidence in mathematics. Do not trust own ability to think.

Belief that math in not so relevant in their field, or at least not for them.

Many students know more math than they have the ability to use.

A surprising number of fundamental misconceptions and inadequate working practices.

Gaps and misconceptions are revealed and fixed

The course is about understanding the formulas we have previously just used and calculated with.

I have realized that my biggest limitation is that I am not careful enough.

At first it was incredibly frustrating that the problems were sometimes not clearly formulated. The identify-the-algorithm thinking I have used in all other mathematical courses turned out to be useless.

Makes a difference

The main purpose of the course was to show how to use the advanced math we have been forced to learn over the years. That it has not happened before is quite strange.

The course has also developed my creative thinking with respect to mathematics, which for me was an entirely new dimension of mathematics.

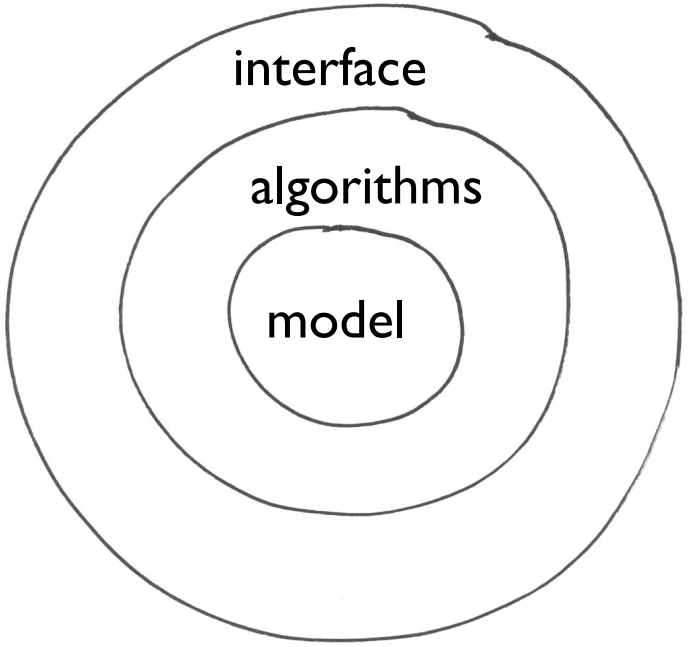
From problem to solution rather than from formula to solution.

It may sound like a cliche but it is not an understatement to say that this course has lifted me to an entirely new level.

The best course I studied at Chalmers. Also the course that will be most useful, I believe.

Student evaluation 2011: 4.8 on a scale from 1-5

Mathematics and software



I think that mathematical modelling is a crucial part of a Software Engineers' tool-kit.

What I have learned about structuring my problem solving is something I will use in other subjects, such as programming. I am very grateful for having learned this.

Important for software engineering students' identity as engineers

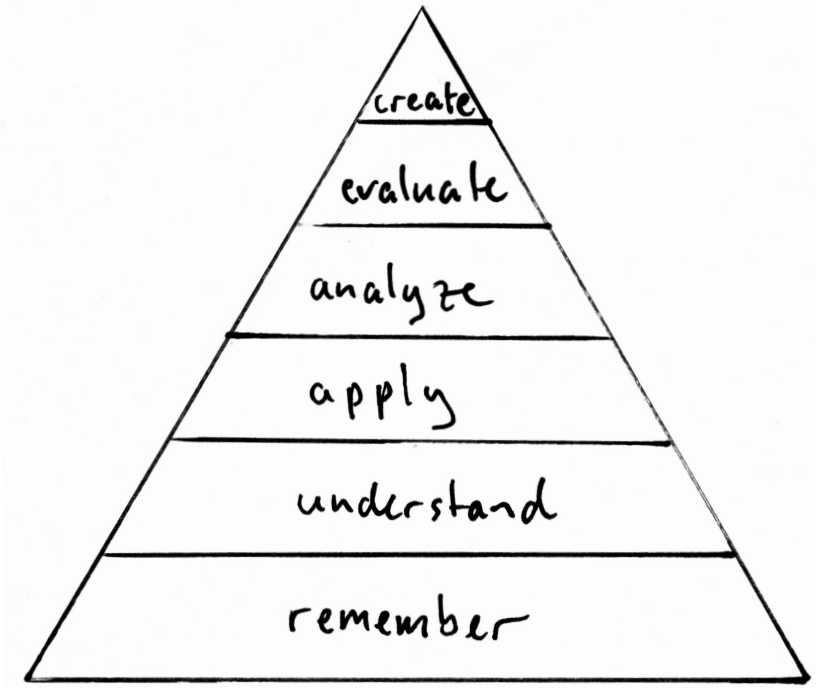
En kurs som
man inte visste
att man saknade.

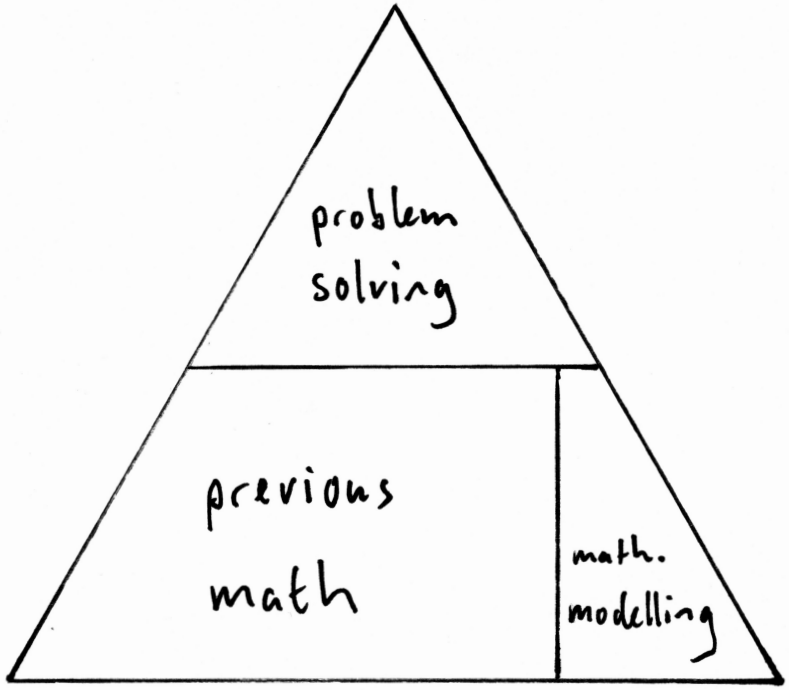
I have developed a more general engineering skill and improved my resourcefulness. And it all has happened so fast that I almost did not think it was true.

[kursen] sätter så att säga ett syfte på den tid, kraft och kostnad man lägger på att lära ut matematik till ingenjörer.

Före kursen var jag orolig för att detta skulle bli ännu en matematikkurs som jag aldrig skulle ha användning för. Det visade sig dock att den oron var obefogad. Den kunskap jag fått är inte bara användbar, den är absolut nödvändig för min utbildning och mitt framtida yrkesliv.

Mathematical modelling and problem solving





Some concluding thoughts

Most important: the course exists!

Very natural to teach mathematical modelling and structured problem solving together.

Intermediate step between traditional courses and projects/PBL.

The effect on the students is remarkable!