Asumptotic Behaviour in Time for a Singular Stochastic Newton Equation

Astrid Hilbert

Christiansen et al. [1], Phys. Rev. E **54**, introduced a focusing formal 2D non-linear Schrödinger equation, perturbed by a damping term, and driven by multiplicative noise. The question in focus is whether collapse of the wave function occurs, i.e. whether the width of the wave function vanishes in finite time, while its L^2 value is conserved, which means explosion of solutions. Being a formal equation only, its rigorous meaning would need further discussion in the first place. This difficulty was by-passed in [1] by introducing the following family of trial wave functions:

$$\Psi(u,t) := C\|\Psi(\cdot,0)\|_{L_2}\tfrac{1}{x(t)}f\left(\tfrac{|u|}{x(t)}\right)\exp\left[i\tfrac{\dot{x}(t)|u|^2}{4x(t)}\right],$$

where $f:\mathbf{R}\to(0,\infty)$ is a rapidly decreasing function and x is an unknown stochastic process describing the width of the corresponding non-linear wave function. In order to answer the question in focus a function f and a process x need to be specified to **i**) match the trial functions to the non-linear wave function and **ii**) study whether an appropriate x reached the origin a.s. in finite time. This work answers part **ii**), with a process x satisfying the singular degenerate system of SDE's:

$$dx(t) = y\,dt, \qquad dy(t) = \frac{1}{x^\alpha}dt - \frac{\gamma}{x^{2\beta}}dt + \frac{\sqrt{2T\gamma}}{x^\beta}dW_t,$$

where the physical relevance of the constants may be found in *arXiv:1405.0151v1*. Beyond existence an uniqueness of solutions we focus on the large time asymptotics of the solutions. For the existence proof methods of Cerny and Engelbert are extended to the 2D system.
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