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Optimal Scheduling for Replacing Perimeter Guarding Unmanned Aerial Vehicles

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Abstract

Guarding the perimeter of an area in order to detect potential intruders is an important task in a variety of security-related applications. This task can in many circumstances be performed by a set of camera-equipped unmanned aerial vehicles (UAVs). Such UAVs will occasionally require refueling or recharging, in which case they must temporarily be replaced by other UAVs in order to maintain complete surveillance of the perimeter. In this paper we consider the problem of scheduling such replacements. We present optimal replacement strategies and justify their optimality.

Keywords: scheduling problem; optimal replacement strategies; perimeter guarding; unmanned aerial vehicles.

1 Introduction

To determine how a team of autonomous robots should guard the perimeter of a large area against a potential intruder, we need to answer two questions: How do we *place* the robots, and when do we *replace* them?

The question of placement has already been extensively covered in the literature, with solutions that vary along many distinct dimensions. For example, some placement algorithms guarantee full coverage of an area or its perimeter, while others provide a certain probability of detecting all targets given specific assumptions about those targets and their properties. Along another dimension, *static* placement problems concern determining how to place sensors (not necessarily associated with robots) in fixed locations . This includes the well-known art gallery problem [9] as well as many coverage problems [2, 3]. In contrast, *dynamic* placement problems concern determining suitable movement strategies for robots, which is particularly useful when the number of robots is insufficient for completely covering the desired area. When the target attempts to avoid detection, this turns into a *pursuit/evasion* problem [10, 7, 4, 6, 1, 8]. In this case, the algorithms may also cover the task of *tracking* an intruder once it has been detected.

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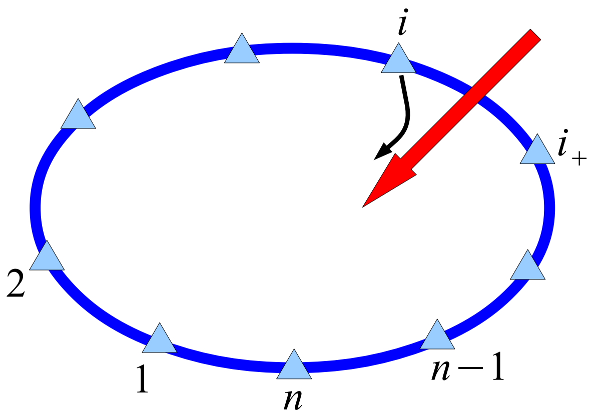

Figure 1: Perimeter guarding UAVs; an intrusion of the perimeter segment between UAVs i and its adjacent UAV i_+

In this paper we consider the second question, that of *replacing* UAVs. This is an essential issue in surveillance applications, since the duration of such missions often exceeds the battery or fuel capacity of the individual UAVs involved, and it is equally relevant regardless of whether the placement of UAVs is nominally static or dynamic.

It should be noted that for the purpose of UAV replacement strategies, there is no significant difference between charging batteries, automatically replacing batteries [11] or refueling with liquid fuel. The main difference is the time required before the UAV can once again participate in a mission. This also depends strongly on the distance to a recharging or refueling station and is in all cases too long for a location to be left unguarded. For brevity, we will therefore limit the discussion below to the use of batteries and battery charge, without loss of generality.

Given that no location can be left unguarded, each UAV that leaves the surveillance mission must at least temporarily be substituted with another. How to schedule such replacements depends strongly on the exact task at hand. For example, a solution adapted to missions where UAVs must follow known trajectories through space [5] is not necessarily suitable or optimal for a set of stationary UAVs guarding a perimeter.

We focus on a general case where n *guarding* UAVs are involved in a perimeter guarding mission. Thus, the guarding UAVs are assumed to be fully charged at the initial time, $t = 0$. Similarly, each replacement UAV is assumed to be fully charged at the time it replaces a guarding UAV. The replacements are assumed to be performed one by one every fixed time interval τ.

When a perimeter intrusion is identified in the area between a pair of adjacent UAVs, like in Fig. 1, the one with the higher battery charge, say UAV i, will attempt to follow and track the intruder. The number of guarding UAVs is assumed to be such that the remaining $n-1$ UAVs are able to continue guarding the entire perimeter.

Any pair of adjacent UAVs is characterized by the higher battery charge of the two UAVs. It is called the pair's *tracking charge*. The lowest tracking charge over all adjacent pairs and all time is called the *critical tracking charge*, and the corresponding pair of adjacent UAVs is called the *weakest*. Another important characteristic of the replacement strategy is the minimal charge over all guarding UAVs, which is called the *critical guarding charge*. The replacement strategy must be constructed so that this charge is always sufficient to allow the corresponding UAV to come back to the base for recharging.

Since it is not a priori known which pair of UAVs may be affected by an intrusion, it is natural to require from the replacement strategy that it maximizes the critical tracking charge and keeps the critical guarding charge at the admissible level at all time. This is the main problem that we address in this paper.

1.1 Organization

The paper is organized as follows. In Section 2, the scheduling problem is formulated. Periodic strategies play an important role in our development of optimal strategies. Their useful properties are considered in Section 3. In Section 4, we present optimal replacement strategies and justify their optimality. In Section 5, we draw conclusions and discuss future work.

2 Problem formulation

The guarding UAVs are assumed to be numbered as shown in Fig. 1 where the notation

$$i_+ = \begin{cases} i+1, & \text{if } i \neq n \\ 1, & \text{if } i = n \end{cases}$$

is used. Let $N = \{1, 2, \ldots, n\}$ stand for the set of all guarding UAVs. We refer to the pair of adjacent UAVs (i, i_+) as pair i. Let i_- denode a number such that $(i_-)_+ = i$.

The battery charge $l_i(t)$ of UAV i is assumed to decrease linearly with time as follows

$$l_i(t) = L - c(t - t'),$$

where the positive scalars L and c denote the full battery charge and discharge rate, respectively, and t' stands for the latest time, before t, when UAV i was replaced. An example of such a function is presented by Fig. 2.

The tracking charge introduced in the previous section is computed for pair i by the formula

$$\bar{l}_i(t) = \max\{l_i(t), l_{i_+}(t)\}.$$

This function is determined by a chosen replacement strategy. It has a form similar to $l_i(t)$:

$$\bar{l}_i(t) = L - c(t - t') \tag{1}$$

with the difference that t' here stands for the latest time before t when any UAV in pair i was replaced.

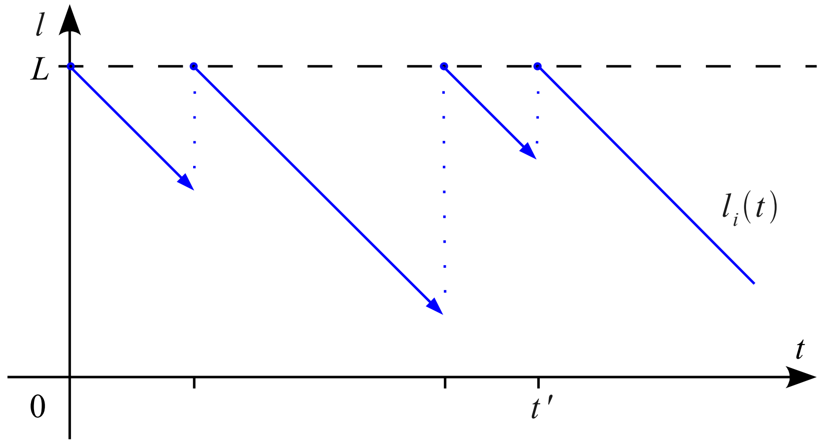

Figure 2: Charge of UAV i vs. time

Any replacement strategy s can be presented as a sequence of integer numbers $v_1, v_2, v_3, \ldots$ from the set N. Each number v_k indicates that UAV v_k should be replaced at time

$$t_k = k\tau, \quad k = 1, 2, 3, \ldots. \tag{2}$$

The set of all replacement strategies is denoted by S. This is actually the set of all infinite sequences of numbers from N.

The critical tracking charge introduced in the previous section is defined as

$$\bar{l}(s) = \inf_{t \geq 0} \min_{i \in N} \bar{l}_i(t). \tag{3}$$

The critical guarding charge is computed by the formula

$$\underline{l}(s) = \inf_{t \geq 0} \min_{i \in N} l_i(t). \tag{4}$$

We obviously have $\underline{l}(s) \leq \bar{l}(s)$ because $l_i(t) \leq \bar{l}_i(t)$ for all $t \geq 0$ and $i \in N$.

As was mentioned in Section 1, the critical guarding charge should be above a given charge level, denoted here by l_{min}, which is sufficient to allow any guarding UAV to come back to the base for recharging. The replacement scheduling problem outlined in Section 1 can now be formulated as follows:

$$\max_{s \in S} \{\bar{l}(s) : \underline{l}(s) \geq l_{min}\}. \tag{5}$$

Since, at any interval $[t_k, t_{k+1})$, the function $\min\{\bar{l}_i(t) : i \in N\}$ decreases linearly with t, formula (3) can be written as

$$\bar{l}(s) = \Lambda(s) - c\tau, \tag{6}$$

where

$$\Lambda(s) = \min_{k \geq 1, i \in N} \bar{l}_i(t_k).$$

Similarly, we can rewrite formula (4) as

$$\underline{l}(s) = \lambda(s) - c\tau, \tag{7}$$

where

$$\lambda(s) = \min_{k\geq 1, i\in N} l_i(t_k).$$

Relations (6) and (7) allow us to present problem (5) in the following equivalent form:

$$\max_{s\in S}\{\Lambda(s) : \ \lambda(s) \geq l_{min} + c\tau\}. \tag{8}$$

Note that only the discrete time values $l_i(t_k)$ are involved in this problem formulation. We use this property in the next two sections for developing replacement strategies and justifying their optimality.

3 Periodic strategies

We call strategy $s = \{\nu_1, \nu_2, \nu_3, \ldots\} \in S$ *periodic* if the segment $\{\nu_1, \nu_2, \ldots, \nu_n\}$ of this sequence is a permutation of the sequence $\{1, 2, \ldots, n\}$ and

$$\nu_{k+n} = \nu_k, \quad \forall k \geq 1.$$

Our analysis of such strategies will be based on the fact that $l_i(t)$ and $\bar{l}_i(t)$ are periodic functions with the period $T = n\tau$.

Periodic strategies play an important role in our development of optimal strategies. One of their key properties is that $\underline{l}(s)$ attains its maximal value all over $s \in S$ if and only if s is periodic. It is an implication of the following result.

Lemma 1 *If $s \in S$ is a periodic strategy, then $\underline{l}(s) = L - cT$. If $s \in S$ is not periodic, then $\underline{l}(s) < L - cT$.*

Proof. The equality $\underline{l}(s) = L - cT$ immediately follows from the fact that, for any periodic strategy s, each guarding UAV is replaced once every fixed time interval T.

Consider any $s \in S$ which is not periodic. This means that, for this strategy, there exists $k \geq 1$ such that $\nu_k \neq \nu_{k+n}$. Suppose, on the contrary, that

$$\underline{l}(s) \geq L - cT. \tag{9}$$

To meet this requirement, every number $i \in N$ should appear at least twice in the sequence of $2n$ numbers $\nu_k, \nu_{k+1}, \ldots, \nu_{k+2n-1}$. Moreover, it should appear exactly twice because N is composed of n numbers. Inequality (9) implies that there should exist $m \leq n$ such that $\nu_k = \nu_{k+m}$. Since $\nu_k \neq \nu_{k+n}$, we have $m < n$. Therefore, the number ν_k appears twice in the first half of the mentioned sequence of $2n$ numbers. Then it does not appear in the second half of the sequence and, for this reason,

$$\underline{l}(s) \leq l_{\nu_k}((k + 2n - 1)\tau) < L - cT.$$

This contradicts assumption (9) and accomplishes the proof of lemma. □

This result allows us to draw a practical conclusion about the admissible time interval between two sequential UAV replacements. From now on, we assume that

$$\tau \geq \frac{L - l_{min}}{cn} , \tag{10}$$

because otherwise problem (5) would not have any feasible solution.

Let a periodic sequence $s = \{v_1, v_2, v_3, \ldots\} \in S$ be given. Then, for any $i \in N$, there exists a unique integer k such that $1 \leq k \leq n$ and $v_k = i$. We denote this dependence of k on i by $\kappa_s(i)$. We shall also use the notations

$$\kappa_s^{min}(i) = \min_{j \in \{i, i_+\}} \kappa_s(j), \quad \kappa_s^{max}(i) = \max_{j \in \{i, i_+\}} \kappa_s(j),$$

$$i_s^{min} = \arg \min_{j \in \{i, i_+\}} \kappa_s(j), \quad i_s^{max} = \arg \max_{j \in \{i, i_+\}} \kappa_s(j).$$

For the problem in focus, all these notations refer to the initial period of time $[0, T]$. In particular, UAV i is replaced for the first time at time $t_{\kappa_s(i)}$. The UAVs i_s^{min} and i_s^{max} of pair i are replaced for the first time at times $t_{\kappa_s^{min}(i)} < t_{\kappa_s^{max}(i)}$.

To justify the optimality of our periodic strategies proposed in the next section, we shall use the following result.

Lemma 2 *Suppose $s \in S$ is a periodic strategy. Then*

$$\Lambda(s) = L - c\tau \max_{i \in N} K_i(s), \tag{11}$$

where

$$K_i(s) = \max\{n + \kappa_s^{min}(i) - 1 - \kappa_s^{max}(i),\ \kappa_s^{max}(i) - 1 - \kappa_s^{min}(i)\}.$$

Proof. Observe that

$$\Lambda(s) = \min_{i \in N} f_i(s),$$

where the objective function

$$f_i(s) = \min_{k \geq 1} \bar{l}_i(t_k). \tag{12}$$

Since $\bar{l}_i(t_k + T) = \bar{l}_i(t_k)$ for all $t_k \geq \kappa_s^{min}(i)\tau$, we can reduce minimization in (12) over $k \geq 1$ to minimization over two discrete intervals, namely,

$$[1,\ \kappa_s^{min}(i) - 1] \quad \text{and} \quad [\kappa_s^{min}(i),\ n + \kappa_s^{min}(i) - 1].$$

It can be easily verified that the relations

$$\min_{1 \leq k \leq \kappa_s^{min}(i) - 1} \bar{l}_i(t_k) \geq L - c\tau K_i(s)$$

and

$$\min_{\kappa_s^{min}(i) \le k \le n+\kappa_s^{min}(i)-1} \bar{l}_i(t_k) = L - c\tau K_i(s)$$

hold for any periodic strategy $s \in S$. Hence, $f_i(s) = L - c\tau K_i(s)$. This implies (11). □

Note that our periodic strategies introduced in the next section admit an easy derivation of $K_i(s)$ and then straightforward calculation of $\Lambda(s)$ by formula (11).

4 Optimal replacement schedule

Before introducing our optimal strategies, we will find an upper bound for the optimal solution $\bar{l}^*$ to problem (5). This result is formulated as follows.

Lemma 3 *Let τ satisfy inequality (10). If n is odd then*

$$\bar{l}^* \le L - \frac{c(n+1)\tau}{2} \,, \tag{13}$$

else

$$\bar{l}^* \le L - \frac{c(n+2)\tau}{2} \,. \tag{14}$$

Proof. Recall that at any moment t_k, the value of $l_i(t_k)$ becomes equal to L for only one $i \in N$, and the battery charge is decreased by $c\tau$ for all other elements of N. This justifies the following property that will be exploited in the proof.

Consider any strategy $s \in S$. Given an integer $m \in [0, n-1]$ and a moment $t_k \ge (m+1)\tau$, the maximal number of elements i in N, for which the inequality

$$l_i(t_k) \ge L - cm\tau \tag{15}$$

holds, is equal to $m+1$. For the rest of elements in N, this inequality is violated. The mentioned maximal number is attained only when there is no repetition in the segment $\{\nu_{k-m+1}, \ldots, \nu_{k-1}, \nu_k\}$ of the sequence s.

Consider, first, the case when n is odd. We shall show that

$$\Lambda(s) \le \Lambda^*_{odd}, \quad \forall s \in S, \tag{16}$$

where

$$\Lambda^*_{odd} = L - \frac{c(n-1)\tau}{2}.$$

Recalling relation (6), one can see that (16) is a stronger result than (13) because no feasibility of s is required here. Suppose, on the contrary to (16), that there exists $s' \in S$ such that

$$\Lambda(s') = L - cm\tau \tag{17}$$

for some integer $m < (n-1)/2$, i.e. $\Lambda(s') > \Lambda^*_{odd}$. As was shown above, at any moment $t_k \geq (n-1)\tau/2$, there exist *at most* $(n-1)/2$ elements i of the set N for which the inequality

$$l_i(t_k) > \Lambda^*_{odd} \tag{18}$$

holds. For the rest of the elements in N, i.e. for *at least* $(n+1)/2$ of those in N, inequality (18) does not hold. Each i satisfying (18) ensures for exactly two pairs, namely (i, i_+) and (i_-, i), that their tracking charges $\bar{l}_i(t_k)$ and $\bar{l}_{i_-}(t_k)$ are strictly above Λ^*_{odd}. Then the same holds for at most $n-1$ pairs in total. Since the total number of all pairs is n, there exists at least one pair, say i', such that $\bar{l}_{i'}(t_k) \leq \Lambda^*_{odd}$. This contradicts the assumption that $\Lambda^*_{odd} < \Lambda(s')$ and proves that (16) holds which, in turn, implies the validity of the upper bound in (13).

Consider now the case of even values of n. Let

$$S_f = \{s \in S : \lambda(s) \geq l_{min} + c\tau\}$$

denote the feasible set in (8). Inequality (10) implies that S_f is not empty. Note that (14) is equivalent to the relation

$$\Lambda(s) \leq \Lambda^*_{even}, \quad \forall s \in S_f, \tag{19}$$

where

$$\Lambda^*_{even} = L - \frac{cn\tau}{2}.$$

Suppose, on the contrary to (19), that there exists a feasible strategy $s' \in S_f$ such that (17) holds for some integer $m < n/2$. This would mean that $\Lambda(s') > \Lambda^*_{even}$. Then, at any moment $t_k \geq n\tau/2$, there must exist at most $n/2$ elements i of the set N for which the inequality

$$l_i(t_k) > \Lambda^*_{even} \tag{20}$$

holds. On the other hand, this amount of elements can not be less than $n/2$, because otherwise there would exist at least one pair, say, i' such that $\bar{l}_{i'}(t_k) \leq \Lambda^*_{even}$. Thus, (20) must hold for exactly $n/2$ elements, which means that only $m = (n-2)/2$ is to be considered. To avoid the existence of the mentioned pair i', these $n/2$ elements must all be either odd, or even. Without loss of generality, we assume that it is the subset of all odd numbers in N for which (20) is satisfied. Then it is the subset of all even numbers in N for which (20) is violated. Let i be odd for which (15) holds as equality. Then $l_i(t_{k+1})$ must become equal to L, because otherwise at least one of the pairs (i_-, i) and (i, i_+) would break the assumption (17). This means that the strategy s' is such that only odd UAVs are replaced. Consequently, at a certain moment after t_k, one of the even vehicles should violate the constraint in problem (8). This contradicts the assumption that s' is feasible and proves that (19) holds, which implies the validity of the upper bound in (14). □

It will later be shown that bounds (13) and (14) are tight.

For the odd values of n, we suggest a strategy denoted by s_{odd} and defined by the recursive formula:

$$\begin{aligned} &\nu_1 = 1, \\ &\nu_k = \begin{cases} \nu_{k-1} + 2, & \text{if } \nu_{k-1} \leq n-2, \\ 1, & \text{if } \nu_{k-1} = n-1, \\ 2, & \text{if } \nu_{k-1} = n, \end{cases} \qquad k = 2, 3, 4, \dots . \end{aligned} \tag{21}$$

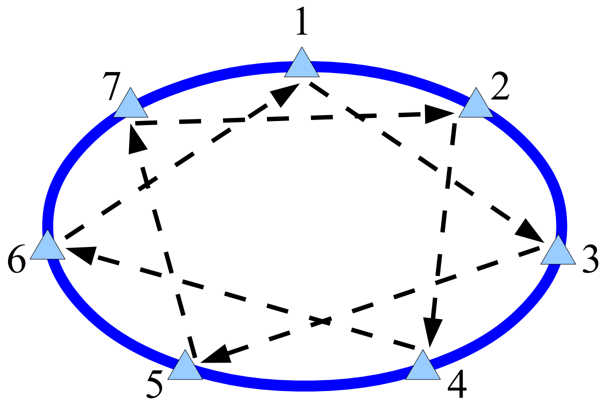

Figure 3: Optimal replacement sequence for 7 UAVs

If, for example, $n = 7$ it produces the UAV replacement sequence

$$\{1,3,5,7,2,4,6,1,3,5,7,2,4,\ldots\}$$

(see Fig. 3). The following result summarizes the main properties of strategy (21).

Theorem 4 *Let n be an odd number. Suppose that τ satisfies inequality (10). Then strategy (21) is periodic. Moreover, it is an optimal solution of problem (5), and the optimal value of the objective function in this problem is*

$$\bar{l}^*_{odd} = L - \frac{c(n+1)\tau}{2} \,. \tag{22}$$

Proof. It can be easily verified that strategy (21) is periodic. Then due to assumption (10) and by Lemma 1, this strategy is feasible in problem (5).

We will prove now that the objective function value $\bar{l}(s_{odd})$ is the same as $\bar{l}^*_{odd}$ in (22). In view of (6), it is sufficient to show that

$$\Lambda(s_{odd}) = L - \frac{c(n-1)\tau}{2} \,. \tag{23}$$

Our proof of this relation is based on the observation that

$$\kappa_{s_{odd}}(i) = \begin{cases} (i+1)/2, & \text{if } i \text{ is odd,} \\ (i+n+1)/2, & \text{if } i \text{ is even,} \end{cases}$$

$$i^{min}_{s_{odd}} = \begin{cases} i, & \text{if } i \neq n \text{ is odd,} \\ 1, & \text{if } i = n, \\ i+1, & \text{if } i \text{ is even,} \end{cases} \qquad i^{max}_{s_{odd}} = \begin{cases} i+1, & \text{if } i \neq n \text{ is odd,} \\ n, & \text{if } i = n, \\ i, & \text{if } i \text{ is even.} \end{cases}$$

This gives

$$\kappa^{min}_{s_{odd}}(i) = \begin{cases} (i+1)/2, & \text{if } i \neq n \text{ is odd,} \\ 1, & \text{if } i = n, \\ (i+2)/2, & \text{if } i \text{ is even,} \end{cases} \qquad \kappa^{max}_{s_{odd}}(i) = \begin{cases} (i+n+2)/2, & \text{if } i \neq n \text{ is odd,} \\ (n+1)/2, & \text{if } i = n, \\ (i+n+1)/2, & \text{if } i \text{ is even.} \end{cases}$$

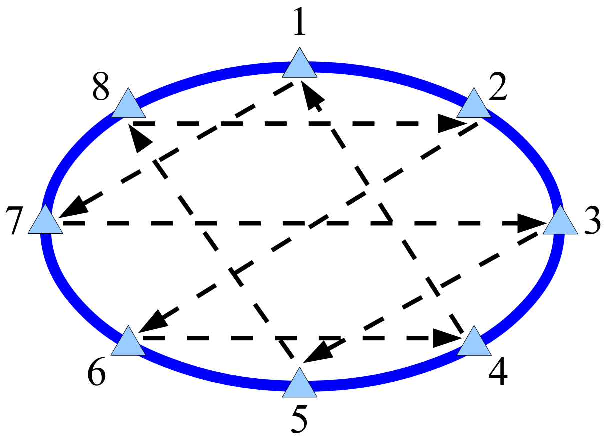

Figure 4: Optimal replacement sequence for 8 UAVs

Consequently,

$$K_i(s_{odd}) = \frac{n-1}{2}, \quad \forall i \in N.$$

Then using Lemma 2, we obtain (23) which implies that $\bar{l}(s_{odd}) = \bar{l}^*_{odd}$.

Observe that $\bar{l}(s_{odd})$ equals the upper bound in (13). Then, by Lemma 3, the strategy s_{odd} solves problem (5). □

It should be mentioned that, by Theorem 4, the proved relation (16) implies that there is no strategy s, even among those infeasible in (8), whose objective function value $\Lambda(s)$ would be better than the one produced by our optimal strategy s_{odd}.

For the even values of n, we suggest a strategy denoted by s_{even} and defined recursively for $k = 1, 2, \ldots, n$ as follows:

$$\begin{array}{l} v_1 = 1, \; v_2 = n-1, \; v_{n/2+1} = n, \; v_{n/2+2} = 2, \\ v_k = \begin{cases} v_{k-2} - (-1)^k 2, & k = 3, 4, \ldots, n/2, \\ v_{k-2} + (-1)^{k-n/2} 2, & k = n/2+2, n/2+3, \ldots, n. \end{cases} \end{array} \tag{24}$$

This sequence is obviously a permutation of $\{1, 2, \ldots, n\}$. The whole sequence v_k is obtained by periodically extending sequence (24). If, for example, $n = 8$ it produces the replacement sequence

$$\{1, 7, 3, 5, 8, 2, 6, 4, 1, 7, 3, 5, 8, 2, 6, 4, \ldots\}$$

(see Fig. 4). The following result summarizes the main properties of the presented strategy.

Theorem 5 *Let n be an odd number. Suppose that τ satisfies inequality (10). Then strategy s_{even} is an optimal solution of problem (5), and the optimal value of the objective function in this problem is*

$$\bar{l}^*_{even} = L - \frac{c(n+2)\tau}{2}. \tag{25}$$

Proof. By reasoning similar to that in the proof of Theorem 4, we shall first show that

$$\Lambda(s_{odd}) = L - \frac{cn\tau}{2} \ . \tag{26}$$

Based on (24), one can easily derive the relations

$$\kappa_{s_{even}}(i) = \begin{cases} i, & \text{if } i \text{ is odd and } 1 \le i \le n/2, \\ n-i+1, & \text{if } i \text{ is odd and } n/2 < i < n, \\ n/2+i, & \text{if } i \text{ is even and } 2 \le i \le n/2, \\ 3n/2-i+1, & \text{if } i \text{ is even and } n/2 < i \le n, \end{cases}$$

$$i^{min}_{s_{even}} = \begin{cases} i, & \text{if } i \text{ is odd}, \\ i_-, & \text{if } i \text{ is even}, \end{cases} \qquad i^{max}_{s_{even}} = \begin{cases} i_+, & \text{if } i \ne n \text{ is odd}, \\ i, & \text{if } i \text{ is even}. \end{cases}$$

They imply

$$\kappa^{min}_{s_{even}}(i) = \begin{cases} i, & \text{if } i \text{ is odd and } 1 \le i \le n/2, \\ n-i+1, & \text{if } i \text{ is odd and } n/2 < i < n, \\ i-1, & \text{if } i \text{ is even and } 2 \le i < n/2, \\ n-i, & \text{if } i \text{ is even and } n/2 \le i \le n, \\ 1, & \text{if } i = n, \end{cases}$$

$$\kappa^{max}_{s_{even}}(i) = \begin{cases} n/2+i+1, & \text{if } i \text{ is odd and } 1 \le i < n/2, \\ 3n/2-i, & \text{if } i \text{ is odd and } n/2 \le i < n, \\ n/2+i, & \text{if } i \text{ is even and } 2 \le i \le n/2, \\ 3n/2-i+1, & \text{if } i \text{ is even and } n/2 < i < n \\ n/2+1, & \text{if } i = n. \end{cases}$$

Therefore,

$$K_i(s_{even}) = \begin{cases} n/2-1, & \text{if } i = n/2 \text{ or } i = n, \\ n/2, & \text{otherwise}. \end{cases}$$

Then using Lemma 2, we obtain (26) which yields $\bar{l}(s_{even}) = \bar{l}^*_{even}$.

Observe that $\bar{l}(s_{even})$ equals the upper bound in (14). Then, by Lemma 3, the strategy s_{even} solves problem (5). ∎

Note that the presented optimal strategies are not unique. Indeed, if the guarding UAVs are counted counterclockwise, the strategies formally defined by (21) and (24) are, obviously, also optimal. Furthermore, it can be easily verified that, if $\{v_k, v_{k+1}, \ldots\}$ is a subsequence of either s_{odd} or s_{even}, this subsequence is also an optimal strategy. This means that any optimal periodic strategy remains optimal under some invariant changes in circular numbering of UAVs, namely, when the circular direction changes between clockwise and counter-clockwise, and when the numbering is shifted clockwise or counter-clockwise.

5 Conclusions and future work

In this paper we considered the problem of scheduling replacements of UAVs in a perimeter guarding task. The main results are the following. A practical importance of periodic replacement strategies was justified. Based on this result, a minimal time interval between two sequential UAV replacements was derived. Replacement strategies were introduced separately for odd and even number of UAVs, and their optimality was proved.

We plan to study the uniqueness of the introduced replacement strategies, or to be precise, the uniqueness of the corresponding classes of strategies generated by the invariant transformations discussed at the end of the previous section.

Our intention is also to consider the more difficult case of optimally replacing UAVs with individual full battery charge L_i and discharge rate c_i.

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