A Game Theoretic Framework for Incentives in P2P Systems

Chiranjeeb Buragohain, Divyakant Agrawal, Subhash Suri

{chiran,agrawal,suri}@cs.ucsb.edu.

Computer Science Dept

University of California, Santa Barbara

Plan

- Selfishness in P2P systems
- Game theory and Nash equilibrium
- Incentives for P2P systems
- Nash equilibrium analysis
- Simulation and results
- Implications for system architecture

Selfishness in P2P Systems

P2P systems vs ordinary distributed systems

- Administration of each node is under individual control
- Goals of individual participants not the same as the goals of the overall system.

When individual and social welfare does not coincide, *selfish* and *rational* individuals will pursue their own goals at the expense of overall social welfare.

The Free Rider Problem

- *Free Rider* : a user who does not contribute to the system, but reaps benefit from it.
  - Inefficient and unfair

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Ad-hoc networks : participating nodes may selfishly decide not to route packets from other nodes.

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Incentives for Sharing

- Monetary incentives (micropayments) : require extensive infrastructure to track transactions. [a]

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  - Do incentives lead to a desirable social outcome?
  - How does one implement such incentives?

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- Questions we address :
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  - How does one implement such incentives?
- Game theory provides a framework to answer the first question

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Game Theory

Game Theory describes interaction of *selfish* and *rational* individuals.

Two Person Prisoner's Dilemma

A, B	Not Confess	Confess
Not Confess	1, 1	10, 0
Confess	0, 10	8, 8

- Strategy : Not Confess vs Confess
- Payoffs/Utilities : Rewards and punishment

Question: given these strategies and payoffs or utilities, what would a rational selfish person do?

Nash Equilibrium

Nash Equilibrium : strategies for the players such that neither player can improve his payoff by switching strategy unilaterally.

- Might not be optimal. (Confess, Confess) is a Nash equilibrium for Prisoner's dilemma.
- Not always unique. Other arguments may be needed to choose between multiple alternatives.
- Pure vs mixed strategy equilibriums

Our goal : define incentives in terms of a game.

D Fudenberg, J. Tirole, *Game Theory*, MIT Press 1991

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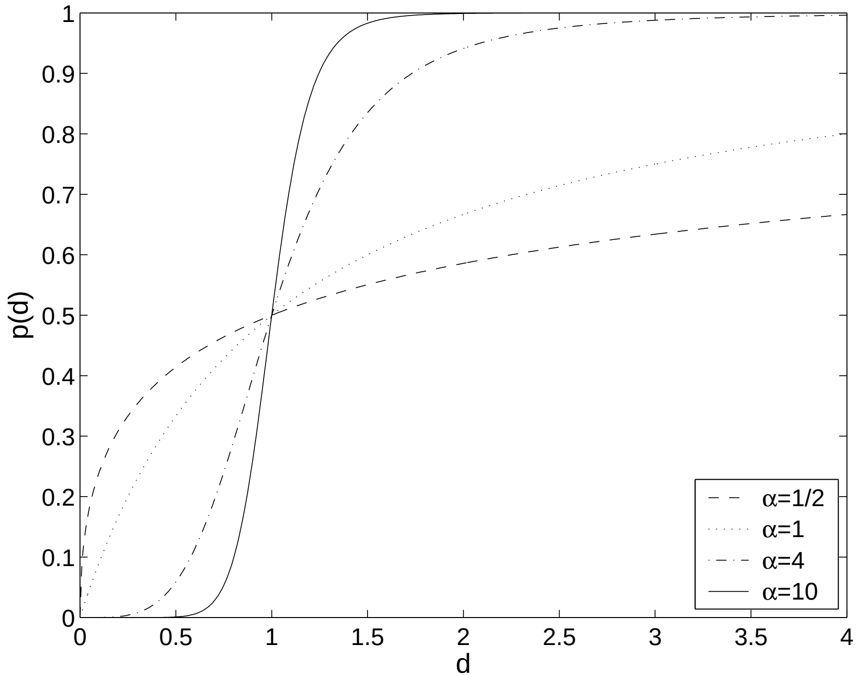
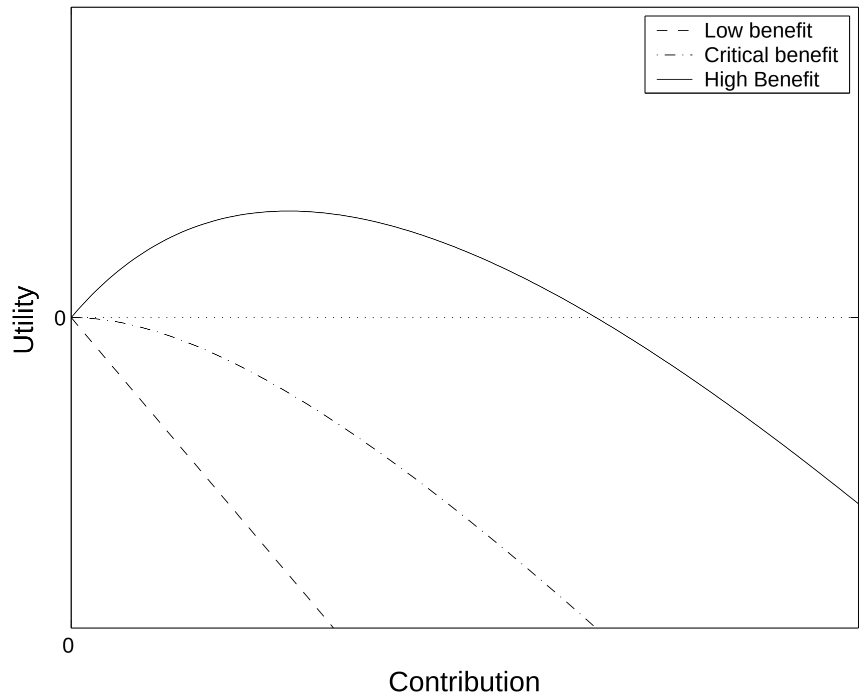
- Total benefit : $b_i = \sum_j b_{ij}$

Incentives and Utility





- Unless b_i exceeds a critical value b_c, the best strategy for the peer i is not to participate!

The Two Person Game

- Easier than solving N person game.
- Insights are applicable to the N person situation as well.

$$u_1 = -d_1 + b_{12} d_2 p(d_1)$$

$$u_2 = -d_2 + b_{21} d_1 p(d_2)$$

Reaction Function: Optimal reaction to the other player's strategy.

$$r_1(d_2) \equiv d_1 = \sqrt{b_{12} d_2} - 1$$

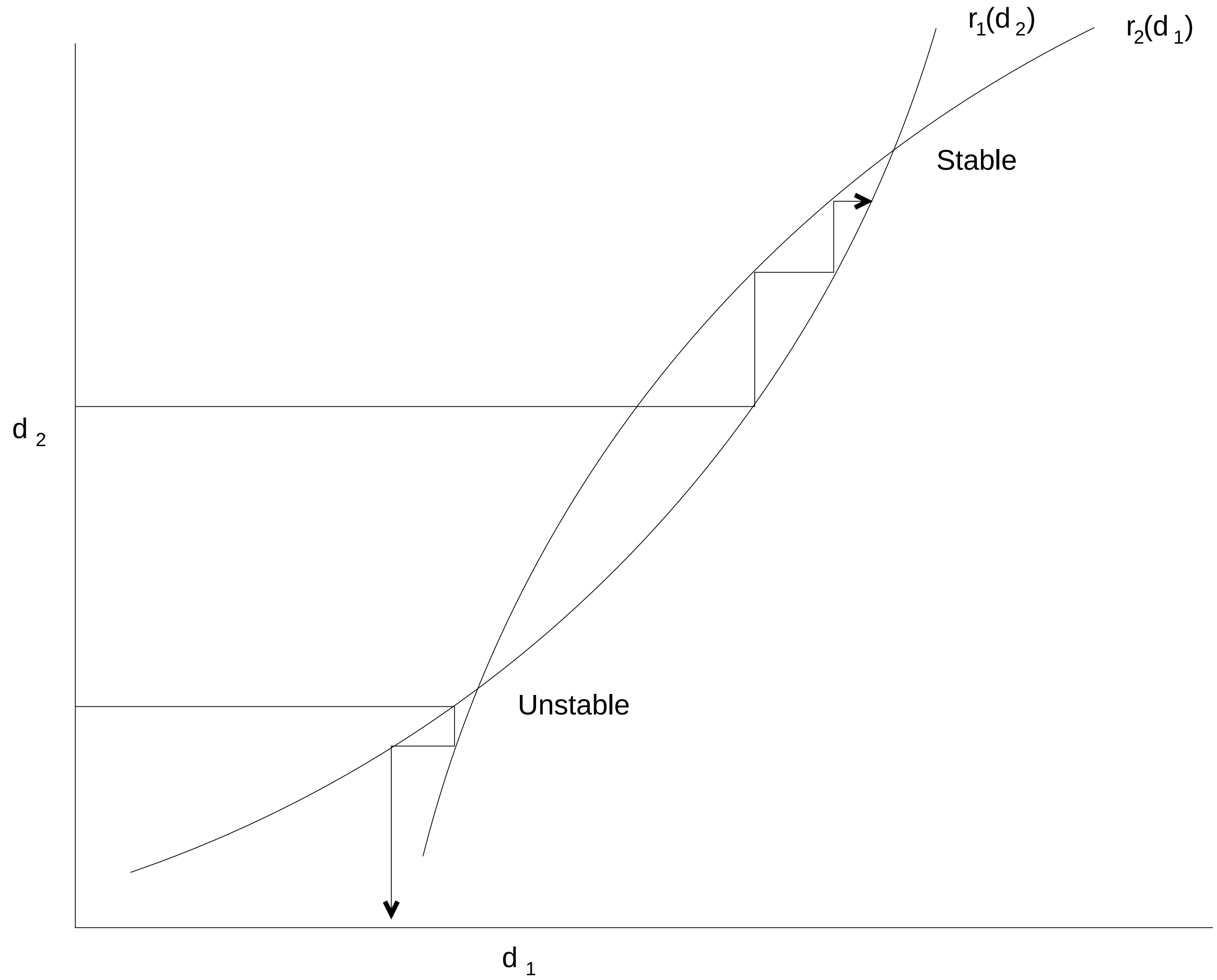
$$r_2(d_1) \equiv d_2 = \sqrt{b_{21} d_1} - 1$$

Fixed Points and Nash Equilibrium

Nash equilibrium is the fixed point.

$$d_1^* = \sqrt{b_{12} d_2^*} - 1$$

$$d_2^* = \sqrt{b_{21} d_1^*} - 1$$



Stability of Nash Equilibrium

Finding Nash equilibrium by iteration :

$$d_1^* = r_1(r_2(r_1(r_2(\ldots))))$$

$$d_2^* = r_2(r_1(r_2(r_1(\ldots))))$$

- Stable equilibrium $\rightarrow$ iteration converges
- Unstable equilibrium $\rightarrow$ iteration diverges

1. Two possible Nash equilibria.
2. The stable equilibrium is also the socially desirable equilibrium.

Homogeneous Two Person Game

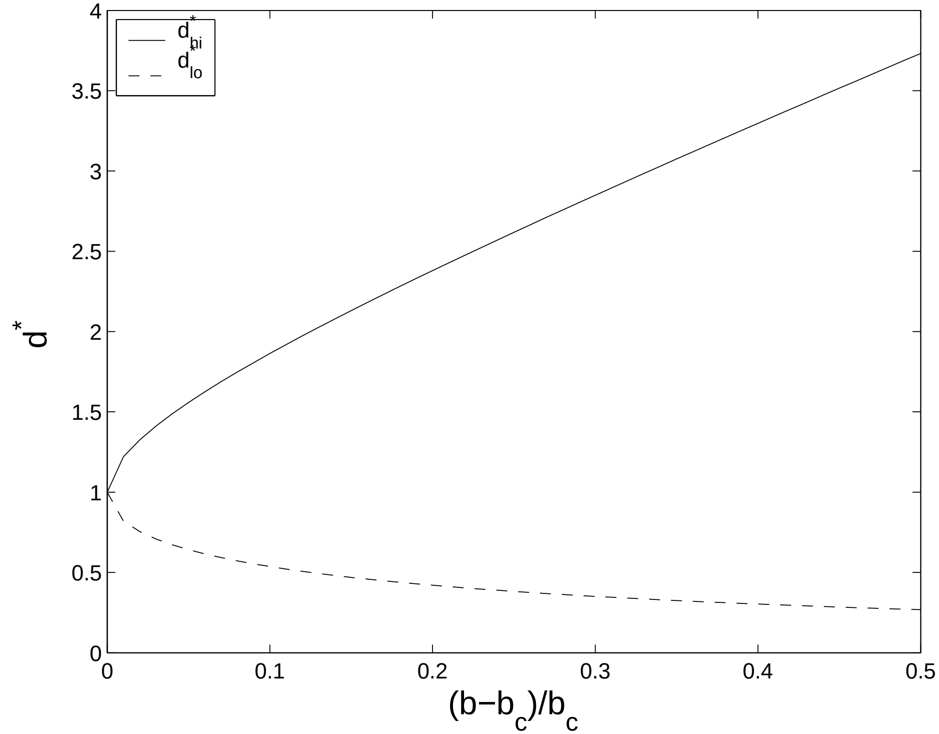
Homogeneous case

$$b_{12} = b_{21} \equiv b$$

$$d_1^* = d_2^* = d^*$$

$$d_{\rm hi}^* \longrightarrow \text{stable}$$

$$d_{\rm lo}^* \longrightarrow \text{unstable}$$



$$d^* = (b/2 - 1) \pm ((b/2 - 1)^2 - 1)^{1/2}$$

No equilibrium for $b < b_c \equiv 4$.

Real World : N Person Game

The fixed point equation

$$d_i^* = \left[\sum_{j \neq i} b_{ij} d_j^* \right]^{1/2} - 1$$

- Nonhomogeneous benefits b_{ij}
- Solve the fixed point problem through an iterative algorithm which mimics real world learning process.
- Compare the solution with the solution for the homogeneous two person game

Finding Nash Equilibrium

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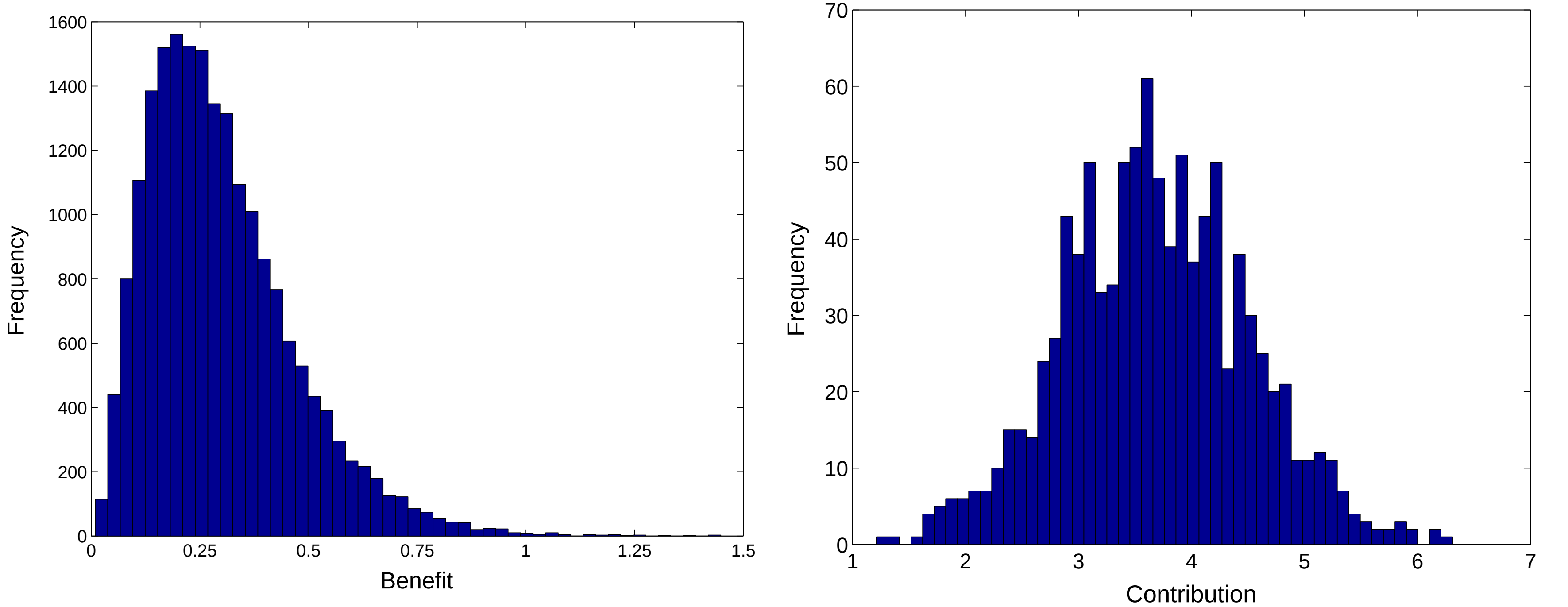
If this algorithm converges, then the set of contributions is a stable Nash equilibrium.

Parameters

- Number of Peers : N = 500 and 1000
- Distribution of b_{ij} : Chosen from a Gamma distribution
- Non-zero b_{ij} : 2% of N
- Initial contributions : d_i chosen from a Gaussian distribution
- $p(d) = d/(1+d)$, i.e. $\alpha = 1$ unless specified otherwise.

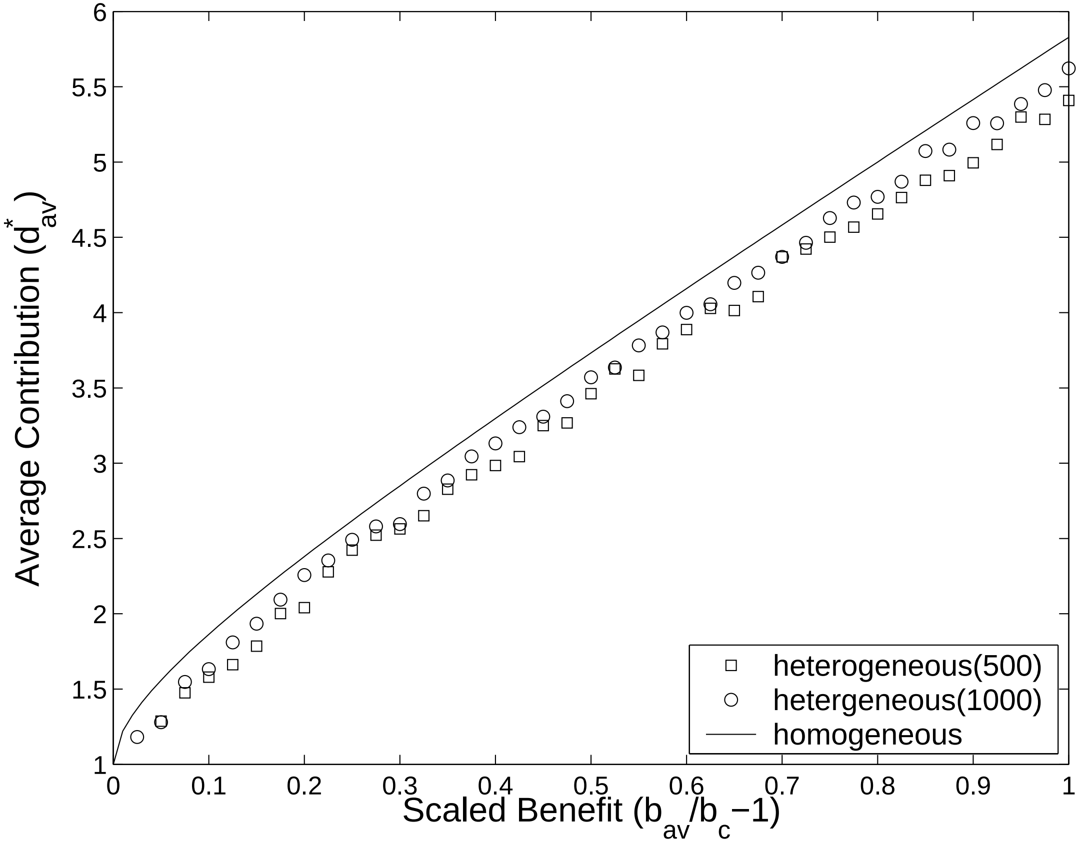
The end results are not sensitive to the precise distribution chosen for b_{ij} or d_i.

Equilibrium Distribution



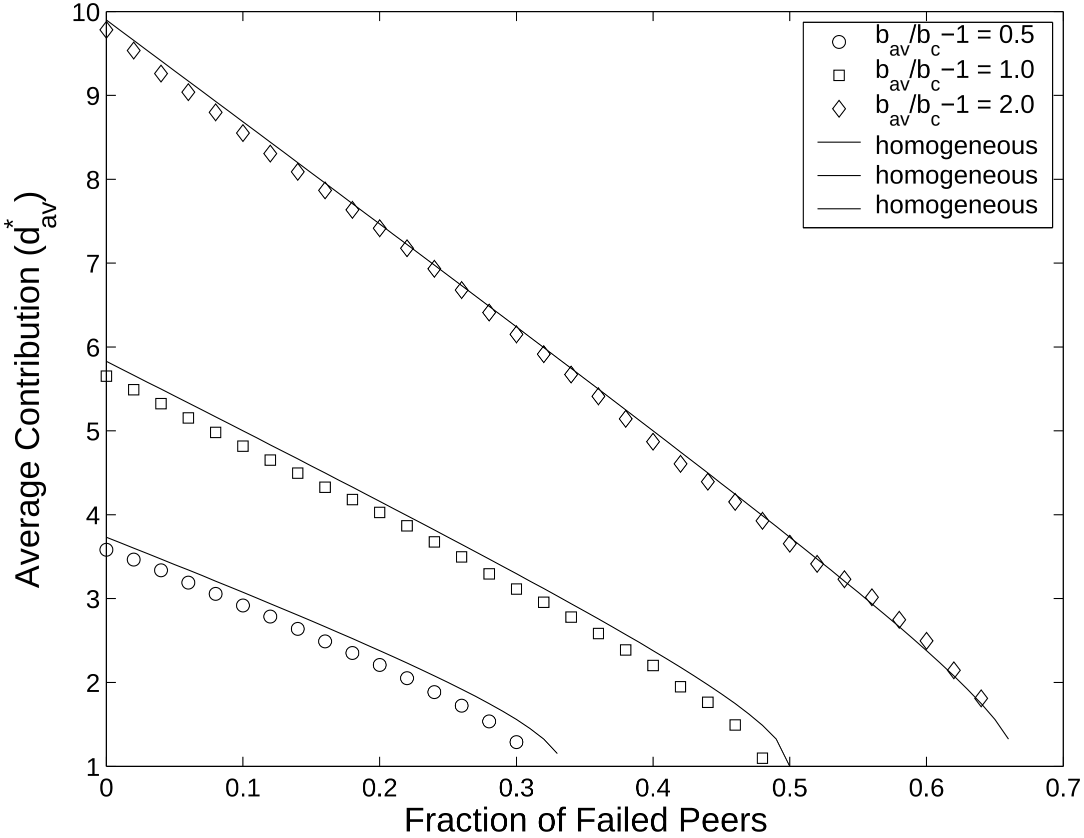
- N = 1000
- Average Benefit $b_{\mathrm{av}} = 6.0$
- Average Contribution $d_{\mathrm{av}} = 3.68$.

Equilibrium for N Person Systems



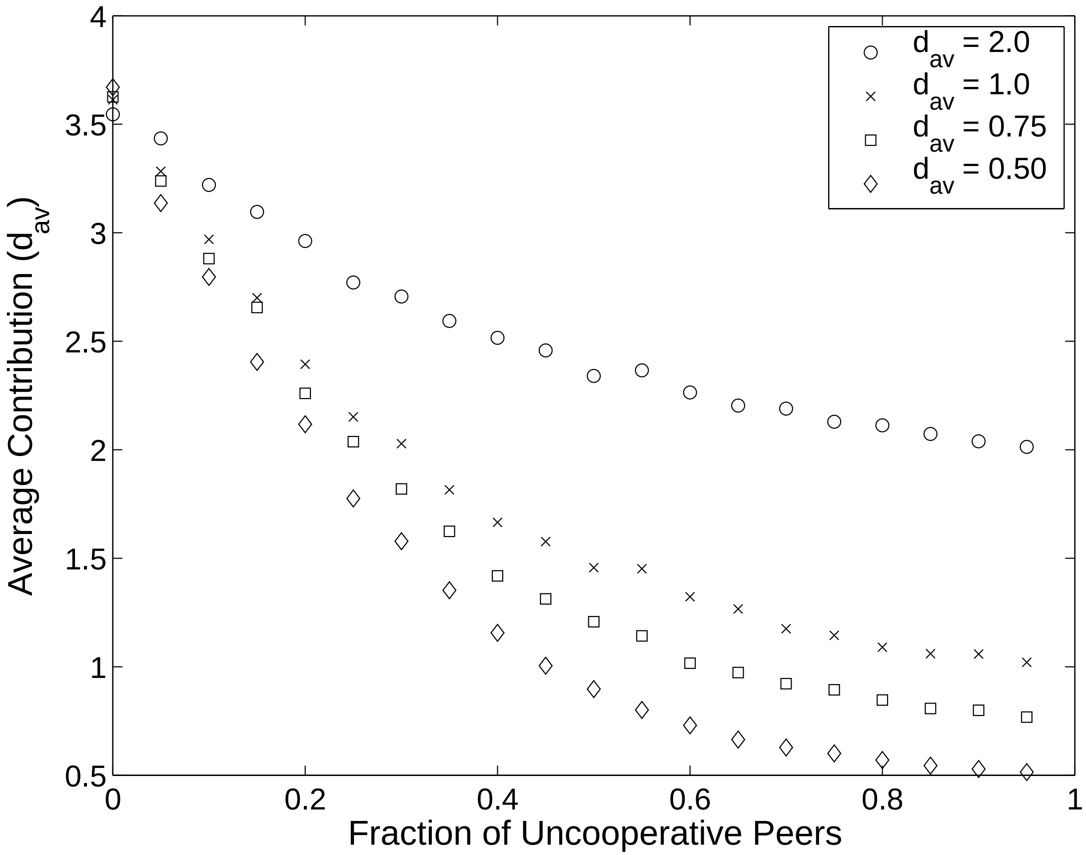
Note that the *average* quantities for the non-homogeneous case corresponds closely with the homogeneous quantities.

Inactive Peers



For $(b_{\rm av} - b_c)/b_c = 2.0$, system can survive even after 2/3rd of the peers have left ($N = 1000$).

Noncooperative Peers



Noncooperative peers are peers who are not rational. They contribute a fixed amount regardless of potential incentives.

Implications for System Architecture

We have proposed differential service based on contribution.

- Contribution
  - Contribution = disk space $\times$ uptime
  - Contribution information is piggybacked with the request for service.
  - Neighbor Audit : Every peer's contribution is monitored by one or more neighbors.

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  - Contribution information is piggybacked with the request for service.
  - Neighbor Audit : Every peer's contribution is monitored by one or more neighbors.
- Incentive by differential service
  - Accept requests from peer with probability $p(d)$.
  - If a request is denied, deny subsequent requests from the same peer for a duration of time

Alternative Metrics

- Contribution
  - Number of uploaded files - number of downloaded files
  - Participation Level of Kazaa = $\frac{\text{uploads}}{\text{downloads}} \times 100$
  - Contribution could be a reputation index like EigenTrust. [a]

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- Incentive
  - Restrict bandwidth to a fraction $p(d)$ of total bandwidth. Kazaa reorders queue.
  - Reduction of search capability by restricting search horizon.

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Conclusion

- Differential service based incentive scheme.
- Socially desirable Nash equilibria exists.
- Nash equilibria are not sensitive to small perturbations.
- The incentive scheme is flexible.
- Easy to implement.