Exploiting Qualitative Spatial Neighborhoods in the Situation Calculus

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Abstract. We present first ideas on how results about qualitative spatial reasoning can be exploited in reasoning about action and change. Current work concentrates on a line segment based calculus, the dipole calculus and necessary extensions for representing navigational concepts like *turn right.* We investigate how its conceptual neighborhood structure can be applied in the situation calculus for reasoning qualitatively about relative positions in dynamic environments.

1 Introduction

Most current reliable robot systems are based on a completely determined geometrical world model. The applied metric methods are tending to fail in frequently changing spatial configurations and when accurate distance and orientation information is not obtainable. Qualitative representation of space abstracts from the physical world and enables computers to make predictions about spatial relations, even when precise quantitative information is not available [1]. Important aspects are topological and positional (orientation and distance) information about in most cases physically extended objects. Calculi dealing with such information have been well investigated over recent years and give general and sound reasoning strategies, e.g. about topology of regions as in RCC-8 [24, 25], about relative position orientation of three points as in Freksa's Double-Cross Calculus [6] or about orientation of two line segments as in the Dipole Calculus [22]. For reasoning with calculi as above standard constraint-based reasoning techniques can be applied. In this paper we investigate conceptual neighborhoods [5]. Two relations are conceptual neighbors if their spatial configurations can be continuously transformed into each other with only minimal change, e.g. in RCC-8 two disconnected regions (configuration 1) cannot overlap (3) without being externally connected (2) in between. Therefore 1 and 2 resp. 2 and 3 are conceptually neighboring relations but not 1 and 3. Expressing these connections between the relations leads to conceptual neighborhood graphs (CNH-graph). For further motivation for qualitative spatial reasoning we refer to [7] .

Frameworks for reasoning about action and change, e.g. the Situation Calculus [19] based programming language Golog [18], also provide facilities for representing and reasoning about sets of spatial locations. Current variants are able to deal with e.g. concurrency [3], continuous change [13] or decision theory [8].

Unfortunately no formal spatial theory, e.g. for relative position terms like left, right or behind, is defined within for dealing with underspecified, coarse knowledge. Therefore every project modeling dynamic domains needs the naive formalization of such a theory by the developer again and again, although such concepts have been formally investigated.

We present first ideas about qualitative navigation on the bases of oriented line segments, which we consider as valuable starting point. In this context we will show one way how the results about conceptual neighborhood can be applied in the situation calculus resp. Golog. In the first stage of this work we will only look at simulated scenarios to omit additional complexity caused by real robot control.

The long term goal is a general representation and usage of qualitative spatial concepts about relative position like e.g. $left$, $right$, or $inFrontOf$ within the situation calculus resp. the programming language Golog, e.g. providing action facilities like $go(leftOf, exhibit_7)$. We do not only expect such formal qualitative concepts being useful in agent programming but also in human machine interaction [31, 20].

In this paper we will present several variants of the Dipole Calculus at different levels of granularity and their corresponding conceptual neighborhoods. We present necessary extensions for expressing robot navigation tasks more adequately. After a brief introduction of the Situation Calculus and the programming language Golog we will present a first approach combining the Dipole Calculus in the Situation Calculus resp. Golog.We will clarify our ideas with concrete examples and end with final conclusions.

2 Qualitative Spatial Reasoning

Qualitative Spatial Reasoning (QSR) is an abstraction that summarizes similar quantitative states into one qualitative characterization. From a cognitive perspective the qualitative method *compares* features of the domain rather than *measuring* them in terms of some artificial external scale [6]. The two main directions in QSR are topological reasoning about regions, e.g. the RCC-8, and positional reasoning about point configurations. An overview is given in [2].

Solving navigation tasks involves reasoning about paths as well as reasoning about configurations of objects or landmarks perceived along the way and thus requires the representation of orientation and distance information [28, 15]. Many approaches deal with global allocentric metrical data. For many navigational tasks we do not need absolute allocentric information about the position, instead we need relative egocentric

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representations and a fast process for updating these relations during navigation [32, 33].

Several calculi dealing with relative positional information have been presented in recent years. Freksa's double cross calculus [6] deals with triples of points and can also be viewed as a positional binary relation between a dipole and a point. Schlieder proposed a calculus based on line segments with clock or counter-clock orientation of generating start and end points in [29]. He presented a CNH-graph of 14 basic relations. Isli and Cohn [14] presented a ternary algebra for reasoning about orientation. This algebra contains a tractable subset of base relations.

Moratz et al. [22] extend Schlieder's calculus. In a first variant ten additional relations are regarded, where the two dipoles meet in one point, resulting in a relation algebra in the sense of Tarski [16] with 24 basic relations. Also an extended version with 69 basic relations is introduced such that spatial configurations can be distinguished in a more fine grained fashion. An application oriented calculus dealing with ternary point configurations (TPCC) is presented in [21]. It is suited for both, human robot communication [20] and spatial reasoning in route graphs [21]. Even more fine grained calculi can be used to do path integration for mobile robots [23]. In [34] a line segment approach for shape matching in a robotic context is presented.

2.1 Neighborhood-based reasoning

Neighborhood-based reasoning describes whether two spatial configurations of objects can be transformed into each other by small changes [5]. The conceptual neighborhood of a qualitative spatial relation which holds for a spatial arrangement is the set of relations into which a relation can be changed with minimal transformations, e.g. by continuous deformation. Such a transformation can be a movement of one object of the configuration in a short period of time. On the discrete level of concepts, neighborhood corresponds to continuity on the geometric or physical level of description: continuous processes map onto identical or neighboring classes of descriptions [7]. Spatial neighborhoods are very natural perceptual and cognitive entities and other neighborhood structures can be derived from spatial neighborhoods, e.g. temporal neighborhoods.

A movement of an agent can then be modeled qualitatively as a sequence of neighboring spatial relations which hold for adjacent time intervals. Using this qualitative representation of trajectories neighborhood-based spatial reasoning can be used as a simple, abstract model of robot navigation and exploration. Neighborhoods can be formed recursively and represented by hierarchical tree or lattice structures.

Schlieder [29] mapped orientation onto ordering. He defined the orientation on triangles and for every set with more than three points recursively for every triangle. He extracted 14 basic relations to reason about ordering of line segments[2]. The conceptual neighborhood graph is shown in Fig. 3. The labels are illustrated in Fig. 4.

From the neighborhood graphs of the individual relations, the neighborhood graph of the overall configuration must be derived. In this global neighborhood graph, spatial transformations from a start state to a goal state can be determined. It has been investigated to use the neighborhood graph of two objects for spatial navigation [29]. It has not been investigated yet how a neighborhood graph for a configuration of more complex or even several objects can be constructed using efficient, qualitative methods based on local knowledge.

A problem for the efficient construction of neighborhood graphs for multiple objects is the combinatorial explosion due to the combined neighborhoods of all objects. A potential solution to this problem is to locally restrict the combination of transitions. If we partition the environment of the moving agent into small parts and then only the neighborhood transition graph for these smaller spatial configurations needs to be considered.

2.2 Dipole Relation Algebra

In [22] a qualitative spatial calculus dealing with two directed line segments, in the following also called *dipole*, as basic entities was presented. These dipoles are used for representing spatial objects with intrinsic orientation. A dipole A is defined by two points, the start point s_A and the end point e_A. The presented calculus deals with the orientation of two dipoles. An example of the relation lrrr is shown in Fig. 1. The four letters denote the relative position (e.g. *left* or *right*) of one of the points to the other dipole:

$$A \text{ lrrr } B := A \text{ l } s_B \wedge A \text{ r } e_B \wedge B \text{ r } s_A \wedge B \text{ r } s_A$$

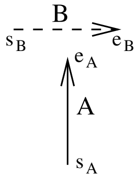

Figure 1. The lrrr orientation relation between two dipoles

Based on a two dimensional continuous space, $\mathbb{R}^2$, the location and orientation of two different dipoles can be distinguished by representing the relative position of start and end points. This means *left* or *right* and the same *start* or *end* point if no more than three points are allowed on a line, and without this restriction *back*, *interior* and *front* additionally (Fig. 2).

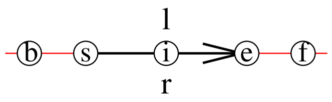

Figure 2. Extended dipole point relations

The first view leads to 24 *jointly exhaustive and pairwise disjoint* (jepd) basic relations, i.e. between any two dipoles exactly one relation holds at any time. Additionally they build

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[2] 16 potential triangle configurations, but two configurations are geometrically impossible.

up a relation algebra ($\mathcal{DRA}_{24}$). A visualization is given in Fig. 4. Because of forming a relation algebra standard constraint-based reasoning techniques can be applied. The unrestricted version leads to a relation algebra with 69 basic relations ($\mathcal{DRA}_{69}$).

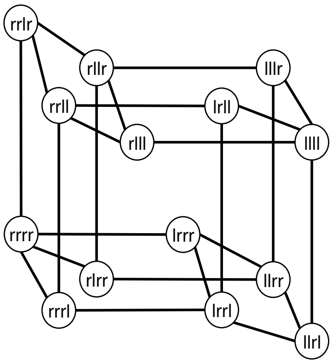

Figure 3. The conceptual neighborhood for the 14 basic relations by Schlieder

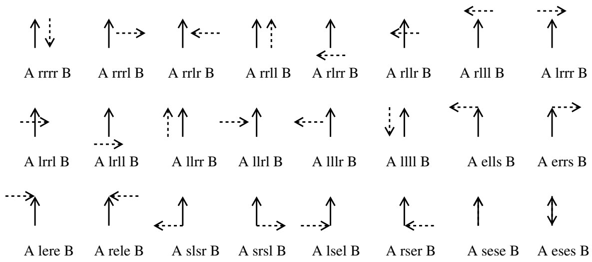

Figure 4. The 24 atomic relations of the dipole calculus. In the dipole calculus orthogonality is not defined, although the visual presentation might suggest.

2.3 Extended Dipole Relation Algebra

Unfortunately $\mathcal{DRA}_{69}$ may not be sufficient for robot navigation tasks, because many different dipole configurations are pooled in one relation. Thus we extend the representation with additional orientation knowledge and derive $\mathcal{DRA}_{77}$.

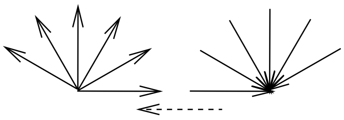

Figure 5. Pairs of dipoles (A: solid, B: dashed) subsumed by the same relation A(rrrr)B

Fig. 5 for example visualizes the large configuration space for the rrrr relation. This might lead to quite squiggly pathes if using these concepts for robot navigation. Other relations being extremely coarse are llrr, rrll and llll. We would expect a more goal directed behavior breaking up the relations by regarding the angle spanned by the two dipoles qualitatively. This gives us an important additional distinguishing feature with four distinctive values. These qualitative distinctions are parallelism (P) or anti-parallelism (A) and mathematically positive and negative angles between A and B, leading to three refining relations for each of the four above mentioned relations (Fig. 6).

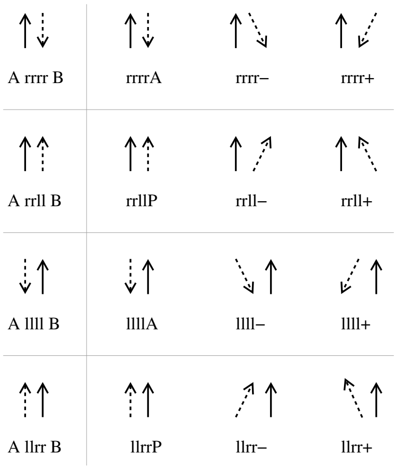

Figure 6. Refined base relations in $\mathcal{DRA}_{77}$

For the other relations a '+' or '−', 'P' or 'A' respectively, is already determined by the original base relation. We give a complete list of the resulting $\mathcal{DRA}_{77}$ algebra:

1. Original relations from$\mathcal{DRA}_{24}$:
   (a) Expanded relations (12): rrrr+, rrrrA, rrrr-, rrll+, rrllP, rrll-, llrr+, llrrP, llrr-, llll+, llllA, llll-
   (b) Unmodified relations (20): rrrl-, rrlr+, rlrr+, rllr+, rlll+, lrrr-, lrrl-, lrll-, llrl-, lllr+ ells+, errs-, lere-, rele+, slsr+, srsl-, lsel-, rser+, seseP, esesA
2. Additional cases on one line[3], seseP and esesA are already defined in 1.(b):
   (a) Basic Allen cases (12): ffbbP, efbsP, ifbiP, bfiiP, sfsiP, beieP, bbffP, bsefP, biifP, iibfP, sisfP, iebeP
   (b) Converse cases relative to Allen (9): ffffA, fefeA, fifiA, fbiiA, fseiA, ebisA, iifbA, eifsA, isebA
3. Other additional cases:
   (a) Without converse (12): lllb+, llfl-, llbr+, llrf-, lirl+, lfrr-, lril-, lrri+, blrr-, irrl-, frrr+, rbrr+
   (b) The converse (12): lbll-, flll+, brll-, rfll+, rlli-, rrlf+, illr+, rilr-, rrbl+, rlir+, rrfr-, rrrb-

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[3] For a relation algebra about this subset of $\mathcal{DRA}_{69}$ see [27].

For lack of space we refer to our web page[4] for the CNH-graphs respectively CNH-tables for $\mathcal{DRA}_{24}$, $\mathcal{DRA}_{69}$ and $\mathcal{DRA}_{77}$. Restricting to relations suited for robotic navigational tasks where dipoles represent solid objects[5] we end up with only 39 base relations, thus giving us a condense CNH-graph.

3 The Situation Calculus

The situation calculus is a second order language for representing and reasoning about dynamic domains. Although many different variants have been developed from the original framework for dealing with e.g. concurrency [3], continuous change [11, 13] or uncertainty [10] all dialects are based on three sorts: *actions*, *situations* and *fluents*.

All changes in the world are caused by an action a_i in the specific situation s_i resulting in the successor situation s_{i+1}. The special constant S_0 denotes the *initial situation* where no action is performed. The binary function $s_{i+1} = do(a_i, s_i)$ starting from S_0 together with a sequence of actions forms a history. Actions are only applicable in the specific situation if preconditions hold which are axiomatized by the special predicate $Poss(a, s) \equiv preconditions$. Fluents are features of the world that might change from situation to situation, e.g. the agents *position* is changed by a *go*-action. Two fluent types can be distinguished. Relational fluents describe truth values while functional fluents hold general values and both might change over situations. They are denoted by predicate resp. function symbols holding the situation as last argument. The action effects on fluents are axiomatized in so called successor state axioms (SSA) [26]. The general form of a SSA for a relational fluent F is

$$\begin{aligned} Poss(a,s) \quad \Rightarrow \quad & (F(.) = true \equiv \\ & a\, makes\, F(.)\, true \\ & \vee F(.) = true\, and\, a\, makes\, no\, change). \end{aligned}$$

With a basic action theory as presented in [17], namely the action precondition axioms, the successor state axioms, the initial situation and an additional unique names axiom a domain model can be formalized.

Golog [18] is a programming language based on the situation calculus for specifying complex tasks like those typically found in robotic scenarios. Golog offers programming constructs well known from imperative programming languages like *sequence*, *if-then-else*,, *while* and *recursive procedures*. Additionally, a *nondeterministic choice* operator is provided to choose from the given alternatives during runtime. Another important difference compared to most other programming languages is the notion of a *test condition*, which in general can be an arbitrary first order sentence.

Golog programs can be viewed as macros for complex actions which are mapped onto primitive actions in the situation calculus. With the above given features Golog serves as integrative framework for programming and planning in deterministic domains. Central for the semantics is the ternary relation $Do(\delta, s, s')$ which is a mapping onto a situation calculus formula. Roughly spoken $Do(\delta, s, s')$ means that given a program δ the situation s' is reachable starting in s. Several extensions e.g. dealing with concurrency [3], sensing [9], continuous change [13], probabilistic projections [12] or decision theory [4, 30] have been presented.

4 Examples

We have presented on the one hand the situation calculus as framework for reasoning about action and change, which spatial relations are build on an absolute geometrical coordinate system. On the other hand we presented the line segment based dipole calculus together with its conceptual neighborhood (CNH) graph for reasoning about relative position. The CNH-graph describes possible qualitative transitions between adjacent relative configurations by continuous motion.

Regarding only two dipoles (compare to Fig. 1 with the dashed dipole representing an agent and the solid dipole a static object) in $\mathcal{DRA}_{24}$ the term *behind* may be defined by relation rlrr and lrll resp. $front$ by rlll and lrrr. In the following we will restrict dipoles to representing only solid objects.

4.1 General Assumptions and Definitions

Below we will use our newly developed dipole calculus $\mathcal{DRA}_{77}$, because we consider $\mathcal{DRA}_{69}$ not being fine grained enough, especially in the context of turning operations. As stated above the CNH-graph is presented on our web page[4]. We define the symmetric binary relation $cnh(p, q)$ holding if two relations p and q are conceptually neighboring. We denote the set of all defined dipoles in the domain with D.

A simple object is a single dipole. A complex object is a polygon, i.e. a sequence of n dipoles $R_i \in D$ where two censecutive dipoles share a common point. For a closed complex object R_0 and R_n must share a common point as well. How such representations can be efficiently and in a compact way be extracted is shown in [34].

Modeling a robot domain in the situation calculus at least one fluent $pos(s)$ for holding the recent position is needed: $pos(s) = \langle r_i, o\rangle$ with $r_i \in \mathcal{DRA}_{77}$ and $o \in O$. In our examples we consider only the basic navigational action $go(r_i, o)$. The precondition that the agent is not blocked holds at any time. Other actions dealing with relative positional information in a domain are e.g. transporting an object R from current position to destination $\langle r_{dest}, O_{dest}\rangle$: $bring(R, r_{dest}, O_{dest})$ or informational questions about spatial configurations.

Because of restricting dipoles to representing only solid objects we can denote subsets (not nessecarily disjoint) of relations suitable for intra-object, agent-object and inter-object relations, regarding a dipole and an object. As defined above the subsequent dipoles of the intra-object description need to share a common point. Therefore only relations containing an e or s are suitable for object descriptions. For the purpose of simplicity we omit the case of an internal connection of two dipoles. Assuming the agent not being allowed to touch any other object only relations without sharing a start, end or internal point are applicable. Thus we can define a subset of relations $\mathcal{DRA}_{77}^{object}$ suitable for intra-object definition.

$$\mathcal{DRA}_{77} \supset \mathcal{DRA}_{77}^{object} = \{\text{ells+,errs-,lere-,rele+,slsr+,srsl-,lsel-,rser+}\}$$

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[4] www.sfbtr8.uni-bremen.de/project/r3/cnh/

[5] Other non solid objects like doorways may also be represented by dipoles.

For agent-object relations all other relations except relations containing an internal dipole connection are suitable, for inter-object relations all $\mathcal{DRA}_{77}$ relations may be used.

4.2 Naive implementation for two dipoles

In a first step we show how a CNH structure might be represented in the situation calculus for two dipoles representing an agent A and an arbitrary object R. The successor state axiom for the *go*-action looks the same as in other domain models without a formal qualitative spatial theory:

$$\begin{aligned} Poss(a,s) \quad \Rightarrow \quad & [pos(do(a,s)) = \langle r_j, o\rangle \equiv \\ & a = go(r_j, o) \vee \\ & [pos(s) = \langle r_j, o\rangle \wedge a \neq go(r_x, o_x)]] \end{aligned}$$

But the graph structure of the dipole calculus helps us for the definition of the preconditions by exploiting the adjacency of the conceptual neighborhood structure. A movement of the agent to a relative position towards the object is only possible if he is already in a conceptually neighboring configuration. This results in:

$$Poss(go(r_j, o), s) \quad \Leftrightarrow \quad pos(s) = \langle r_i, o\rangle \wedge cnh(r_i, r_j).$$

Assuming an agent A and an object R being in relative position $A(\text{lrrr-})B$ with the goal of being $A(\text{ffffA})B$. The situation calculus and CNH-graph will give the same solution, namely two options to go around R. We sketched the action sequence resp. the transition through neighboring CNH-graph states in Fig. 7.

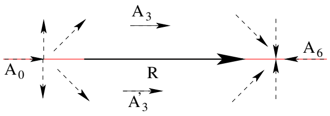

Figure 7. Simple example with two options for Agent A going round object R

4.3 Complex objects (Going round the Kaaba)

Now we present an example for a complex object. One of the tasks during the hadsch (the great Muslim pilgrimage) is rounding the Kaaba (a cubic building in the main mosque in Mekka) seven times. The knowledge about the Kaaba k (compare Fig. 8) can be represented as follows:

$$R_0(errs-)R_1 \wedge R_1(errs-)R_2 \wedge R_2(errs-)R_3 \wedge R_3(errs-)R_0$$

The agent A starts in position A_0 with $A_0(\text{rrllP})R_0$. At this time the other walls of the Kaaba are of no interest for determination of the relative position. Going round the corner of R_0 and R_1 we get the relations shown in Fig. 9.

Looking at all relations for a round trip this repeats for all corners while the other sides provide no useful knowledge.

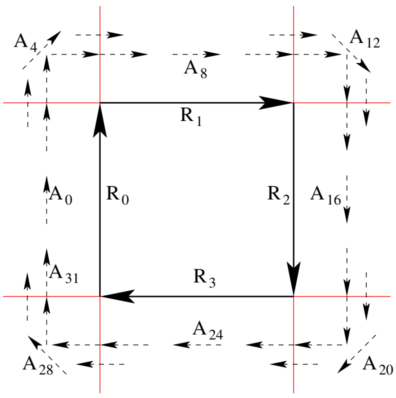

Figure 8. The 32 different qualitative positions of the agent A relative to the kaaba $\{R_0 ... R_3\}$

R_x / A_y	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
R_0	$rrllP$	$rrllP$	$rrllP$	$rrllP$	$rrll+$	$rrlf+$	$rrlr+$	$rrfr+$	$rrrr+$
R_1	$rrrr-$	$rrrb-$	$rrrl-$	$rrbl-$	$rrll-$	$rrllP$	$rrllP$	$rrllP$	$rrllP$

Figure 9. The first nine relations going round the Kaaba with $A_y(r_{y,x})R_x$

Thus in this example only two sides are sufficient for describing the relative position of an agent towards the complete object. We expect this being true for more complex, but convex objects, although we have no formal proof so far.

4.4 Going towards macro definitions

After extracting the neighborhoods for one complex action possibility like "turn right" we are now heading for some sort of macro definition such that an agent is able to perform a "turn right" on the basis of line segments and imprecise orientation information.

Imagine being blind standing at a wall with the task of turning right at the next corner with arbitrary angle and describing it to an external person. The only sensor is one's own right hand extended to the right front which can be seen as some sort of coarse "orientation sensor" transfering the task to a robot. One way describing the process of the first right turn in Fig. 8 might be:

1. Follow the wall until you feel an edge (A_1)
2. Go a little straight ahead so that the edge is to the right of you (A_2), i.e. the next wall comes just into reach on the right side.
3. Turn (in a bow) right around the corner until you are parallel to the next wall (A_3-A_5)
4. Go a little straight ahead until the first wall just gets out of reach (A_6)
5. Go straight ahead until the corner is right behind you (A_7)
6. Follow the wall (A_8)

All the named actions can be modeled as local behaviors and with the help of the base relations presented in the $\mathcal{DRA}_{77}$. If for example loosing parallelity ($A(rrllP)R_0$) to a wall while following it, we have to look whether we have a mathematically positive or negative orientation towards the relating dipole and turn respectively. We will take such descriptions as a basis for our macro definitions. In the first sight the relations of the ($\mathcal{DRA}_{77}$) might seem to be too fine grained to represent a simple behavior like turning right adequately. But without the additional relations compared to the ($\mathcal{DRA}_{24}$) we have not found a way making the transition from one reference dipole to another (from R_0 to R_1) possible, which is necesarry to model going round a corner.

5 Conclusion and Outlook

We presented the design and implementation how the concept of conceptual neighborhood could be exploited for reasoning about relative positional information in the situation calculus in the absence of precise quantitative information. We introduced an extended dipole relation algebra $\mathcal{DRA}_{77}$ better suited for spatial navigation. We expect that every qualitative calculus with a conceptual neighborhood can be translated in a straightforward manner naively onto preconditions and successor state axioms. We have shown by example that not all dipoles of a complex object are necessary to determine the relative position towards the object. We expect the results for connected complex objects being applicable for several not connected dipoles. Additionally we extracted several subsets of the base relations for representing a complex object and dynamic agent behavior.

Future work will deal with the questions how to keep the position representation small for more than one dipole respectively object. A naive implementation would lead to a combinatorial explosion, because the relative position of the agent has to be traced for every single dipole. We will also look on the effects allowing dipoles to represent non-solid entities, e.g. doorways, and potentials to define some sort of general macro definitions for *turnLeft* resp. *turnRight* or *GoAround* by paths in the conceptual neighborhood graph.

6 Acknowledgment

The authors like to thank Diedrich Wolter, Christian Freksa, Jochen Renz and Marco Ragni for fruitful discussions and impulses. And we would like to thank Marc-Björn Seidel for computing the neighborhood graph for different calculi. Our work was supported by the DFG Transregional Collaborative Research Center SFB/TR 8 "Spatial Cognition".

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