Optimization and Parallelization of Sequential Programs

Introduction to Data Dependence Analysis

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Outline

Towards (semi-)automatic parallelization of sequential programs

- Data dependence analysis for loops
  - Dependence tests
- Some loop transformations
  - Loop invariant code hoisting, loop unrolling, loop fusion, loop interchange, loop blocking and tiling, scalar expansion, and more
- Static loop parallelization
- Idiom recognition
- Run-time loop parallelization
  - Doacross parallelization
  - Inspector-executor method
  - If time permits: thread-level speculation

Foundations: Control and Data Dependence

- Consider statements *S, T* in a sequential program (*S*=*T* possible)
  - Scope of analysis is typically a function, i.e. intra-procedural analysis
  - Assume that a control flow path *S* … *T* is possible
  - Can be done at arbitrary granularity (instructions, operations, statements, compound statements, program regions)
  - Relevant are only the read and write effects on memory (i.e. on program variables) by each operation, and the effect on control flow

- **Control dependence** *S* → *T*,
  if the fact whether *T* is executed may depend on *S*
  (e.g. condition)
  - Implies that relative execution order *S* → *T* must be preserved when restructuring the program
  - Mostly obvious from nesting structure in well-structured programs, but more tricky in arbitrary branching code (e.g. assembler code)

Example:

S: **if** (…) {
 …
T: **…**
 …
}

Foundations: Control and Data Dependence

- **Data dependence** *S* → *T*,
  if statement *S* *may* execute (dynamically) before *T*
  and both *may* access the same memory location
  and at least one of these accesses is a write
  - Means that execution order "*S* before *T*" must be preserved when restructuring the program
  - In general, only a conservative over-estimation can be determined statically
  - **flow dependence**: (RAW, read-after-write)
    - *S* may write a location z that *T* may read
  - **anti dependence**: (WAR, write-after-read)
    - *S* may read a location x that *T* may overwrite
  - **output dependence**: (WAW, write-after-write)
    - both *S* and *T* may write the same location

Example:

S: z = … ;
…
T: … = ..z.. ;

(flow dependence)

Dependence Graph

- **(Data, Control, Program) Dependence Graph:** Directed graph, consisting of all statements as vertices and all (data, control, any) dependences as edges.

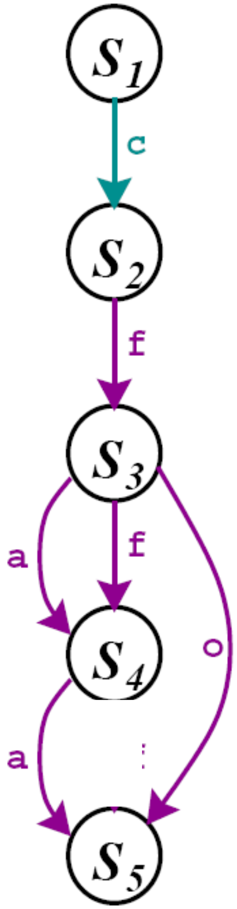
S_1: **if** (e) **goto** S_3

S_2: $a \leftarrow \dots$

S_3: $b \leftarrow a * c$

S_4: $c \leftarrow b * f$

S_5: $b \leftarrow x + f$



control dependence by control flow: $S_1 \delta^c S_2$

data dependence:

flow / true dependence: $S_3\ \delta^f\ S_4$

$S_3 \lhd S_4$ and $\exists b : S_3$ writes b, S_4 reads b

anti-dependence: $S_3\ \delta^a\ S_4$

$S_3 \lhd S_4$ and $\exists c : S_3$ reads c, S_4 writes c

output dependence: $S_3\ \delta^o\ S_5$

$S_3 \lhd S_5$ and $\exists b : S_3$ writes b, S_5 writes b

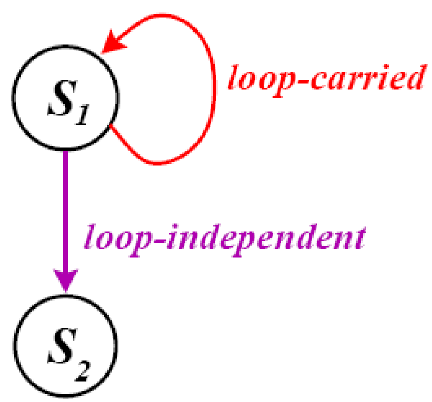
Data Dependence Graph

- **Data dependence graph for straight-line code** (”basic block”, no branching) is always acyclic, because relative execution order of statements is forward only.
- **Data dependence graph for a loop:**
  - Dependence edge *S*→*T* if a dependence may exist *for some pair of instances* (iterations) of *S*, *T*
  - Cycles possible
  - Loop-independent versus loop-carried dependences

Example:

```
   for (i=1; i<n; i++) {
S1:   a[i] = b[i] + a[i-1];
S2:   b[i] = a[i];
   }
```

(assuming that we know statically that arrays a and b do not intersect)



Example

for i **from** 2 **to** 9 **do**

$S_1 \quad X[i] \leftarrow Y[i] + Z[i]$

$S_2 \quad A[i] \leftarrow X[i-1] + 1$

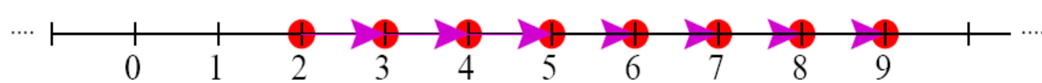
od

(assuming that we statically know that arrays A, X, Y, Z do not intersect, otherwise there might be further dependences)

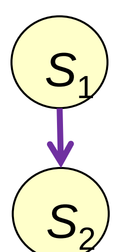
	$i=2$	$i=3$	$i=4$	...
S_1	$X[2] \leftarrow Y[2] + Z[2]$	$X[3] \leftarrow Y[3] + Z[3]$	$X[4] \leftarrow Y[4] + Z[4]$	...
S_2	$A[2] \leftarrow X[1] + 1$	$A[3] \leftarrow X[2] + 1$	$A[4] \leftarrow X[3] + 1$	...

There is a loop-caried, forward, flow dependence from S_1 to S_2.

Iteration space dependence graph:
(Iterations unrolled)



Data dependence graph:



Why Loop Optimization and Parallelization?

Loops are a promising object for program optimizations, including automatic parallelization:

- High execution frequency
  - Most computation done in (inner) loops
  - Even small optimizations can have large impact (cf. Amdahl's Law)
- Regular, repetitive behavior
  - compact description
  - *relatively* simple to analyze statically
- Well researched

Data Dependence Analysis for Loops

A more formal introduction

Christoph Kessler, IDA,
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Data Dependence Analysis – Overview

- Important for loop optimizations, vectorization and parallelization, instruction scheduling, data cache optimizations
- Conservative approximations to disjointness of pairs of memory accesses
  - weaker than data-flow analysis
  - but generalizes nicely to the level of individual array element
- Loops, loop nests
  - Iteration space
  - Array subscripts in loops
  - Index space
- Dependence testing methods
- Data dependence graph
- Data + control dependence graph
  - Program dependence graph

Precedence relation between statements

S_1 statically (textually) precedes S_2 S_1 pred S_2

S_1 dynamically precedes S_2 $S_1 \lhd S_2$

Within loops, loop nests: pred $\neq$ $\lhd$

S_1: $s \leftarrow 0$
for i **from** 1 **to** n **do**
S_2 : $s \leftarrow s + a[i]$
S_3 : $a[i] \leftarrow s$
od

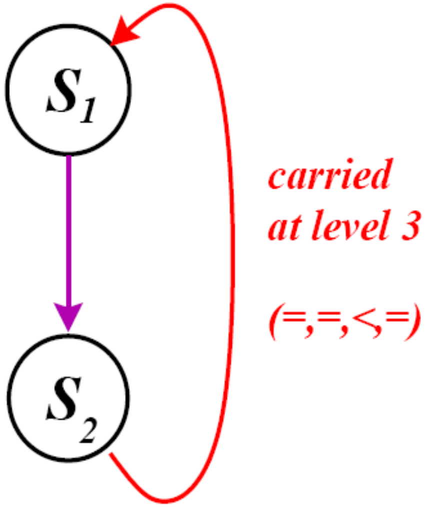
Loop Iteration Space

Beyond basic blocks: pred $\neq \triangleleft$

Canonical loop nest: (HIR code)

for i_1 **from** 1 **to** n_1 **do**
    for i_2 **from** 1 **to** n_2 **do**
        $\ddots$
            for i_k **from** 1 **to** n_k **do**

$$S_1(i_1,...,i_k): \quad A[i_1, 2*i_3] \leftarrow B[i_2, i_3] + 1$$
$$S_2(i_1,...,i_k): \quad B[i_2, i_3+i_4] \leftarrow 2 * A[i_1, 2*i_3]$$



Iteration space: $ItS = [1..n_1] \times [1..n_2] \times ... \times [1..n_k]$

(the simplest case: rectangular, static loop bounds)

Iteration vector $\vec{i} = \langle i_1, ..., i_k \rangle \in ItS$

Example

for i **from** 2 **to** 9 **do**

$S_1 \quad X[i] \leftarrow Y[i] + Z[i]$

$S_2 \quad A[i] \leftarrow X[i-1] + 1$

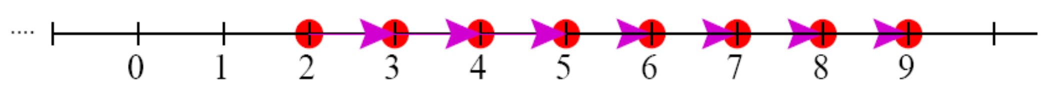
od

(assuming that we statically know that arrays A, X, Y, Z do not intersect, otherwise there might be further dependences)

	$i=2$	$i=3$	$i=4$	...
S_1	$X[2] \leftarrow Y[2] + Z[2]$	$X[3] \leftarrow Y[3] + Z[3]$	$X[4] \leftarrow Y[4] + Z[4]$	...
S_2	$A[2] \leftarrow X[1] + 1$	$A[3] \leftarrow X[2] + 1$	$A[4] \leftarrow X[3] + 1$	...

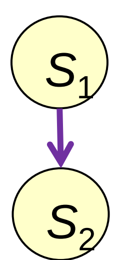
There is a loop-caried, forward, flow dependence from S_1 to S_2.

Iteration space dependence graph:



(Iterations unrolled)

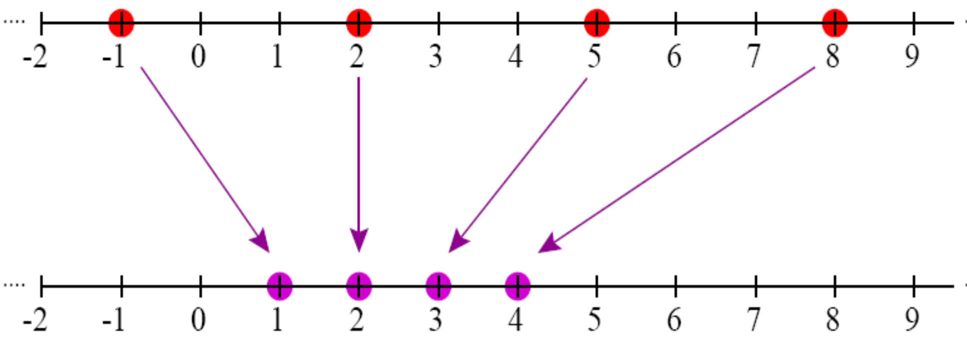
Data dependence graph:



Loop Normalization

Given a loop of the form

for I **from** L **to** U **step** S **do**
… I …
od



normalize the loop:

- lower bound 0 (C) resp. 1 (Fortran)
- step size +1

$\rightarrow$ update all occurrences of the loop counter I by $i*S-S+L$

for i **from** 1 **to** $(U-L+S)/S$ **step** 1 **do**
… $(i*S-S+L)$ …
od

$I \leftarrow i*S-S+L$

Dependence Distance and Direction

Lexicographic order on iteration vectors $\rightarrow$ dynamic execution order:

$S_1(\langle i_1, \ldots, i_k \rangle) \lhd S_2(\langle j_1, \ldots, j_k \rangle)$ iff

either S_1 pred S_2 and $\langle i_1, \ldots, i_k \rangle) \leq_{lex} \langle j_1, \ldots, j_k \rangle$

or $S_1 = S_2$ and $\langle i_1, \ldots, i_k \rangle) <_{lex} \langle j_1, \ldots, j_k \rangle$

distance vector $\vec{d} = \vec{j} - \vec{i} = \langle j_1 - i_1, \ldots, j_k - i_k \rangle$

direction vector $dirv = \mathrm{sgn}(\vec{j} - \vec{i}) = \langle \mathrm{sgn}(j_1 - i_1), \ldots, \mathrm{sgn}(j_k - i_k) \rangle$

in terms of symbols = $< \; > \; \leq \; \geq$ *

Example: $S_1(\langle i_1, i_2, i_3, i_4 \rangle) \; \delta^f \; S_2(\langle i_1, i_2, i_3, i_4 \rangle)$

distance vector $\vec{d} = \langle 0, 0, 0, 0 \rangle$, direction vector $dirv = \langle =, =, =, = \rangle$,

loop-independent dependence

Example: $S_2(\langle i_1, i_2, i_3, i_4 \rangle) \; \delta^f \; S_1(\langle i_1, i_2, i_3 + i_4, i_4 \rangle)$

distance vector $\vec{d} = \langle 0, 0, ?, 0 \rangle$, direction vector $dirv = \langle =, =, >, = \rangle$,

loop-carried dependence (carried by i_3-loop / at level 3)

Dependence Equation System

One-dimensional array A accessed in k nested loops:

$$S_1 : \quad ...A[f(\vec{i})]...$$
$$S_2 : \quad ...A[g(\vec{i})]...$$

Is there a dependence between $S_1(\vec{i})$ and $S_2(\vec{j})$ for some $\vec{i}, \vec{j} \in ItS$?

typically f, g linear: $f(\vec{i}) = a_0 + \sum_{l=1}^{k} a_l i_l,\quad g(\vec{i}) = b_0 + \sum_{l=1}^{k} b_l i_l,$

Exist $\vec{i}, \vec{j} \in \mathbb{Z}^k$ with $f(\vec{i}) = g(\vec{j})$, i.e., $a_0 + \sum_{l=1}^{k} a_l i_l \;=\; b_0 + \sum_{l=1}^{k} b_l j_l,$ dep. equation

subject to $\vec{i}, \vec{j} \in ItS$, i.e.,

$$1 \le i_1 \le n_1, \qquad 1 \le j_1 \le n_1,$$
$$\vdots \qquad\qquad \vdots$$
$$1 \le i_k \le n_k, \qquad 1 \le j_k \le n_k$$

iter. space constraints: linear inequalities

$\Rightarrow$ constrained linear Diophantine equation system $\rightarrow$ ILP (NP-complete)

Linear Diophantine Equations

$$\sum_{j=1}^{n} a_j x_j = c$$

where $n \geq 1, \quad c,\ a_j \in \mathbb{Z}, \quad \exists j:\ a_j \neq 0, \quad x_i \in \mathbb{Z}$

Example 1: $x + 4y = 1$

has infinitely many solutions, e.g. $x = 5$ and $y = -1$.

Example 2: $5x - 10y = 2$

has no solution in $\mathbb{Z}$: absolute term must be multiple of 5

Theorem:

$\sum_{j=1}^{n} a_j x_j = c$ has a solution **iff** $\gcd(a_1, \ldots, a_b) | c$.

Proof: see e.g. [Zima/Chapman p. 143]

Dependence Testing, 1: GCD-Test

Often, a simple test is sufficient to prove independence: e.g.,

gcd-test [Banerjee'76], [Towle'76]:

independence if

$$\gcd\left(\bigcup_{l=1}^{n}\{a_l, b_l\}\right) \nmid \sum_{l=0}^{n}(a_l - b_l)$$

constraints on ItS not considered

Example: **for** i **from** 1 **to** 4 **do**

$$S_1 : \quad b[i] \leftarrow a[3*i-5]+2$$
$$S_2 : \quad a[2*i+1] \leftarrow 1.0/i$$

solution to $2i+1 = 3j-5$ exists in $\mathbb{Z}$ as $\gcd(3,2) | (-5-1+3-2)$

not checked whether such i, j exist in $\{1, ..., 4\}$

For multidimensional arrays?

subscript-wise test vs. linearized indexing

for i ...

$S_1:\ \ldots A[x[i], 2*i]\ldots$

$S_2:\ \ldots A[y[i], 2*i+1]\ldots$

for i ...

$S_1:\ \ldots A[i,i]\ldots$ $\qquad A[i*(s_1+1)]$

$S_2:\ \ldots A[i,i+1]\ldots$ $\qquad A[i*(s_1+1)+1]$

Moreover:

Hierarchical structuring of dependence tests [Burke/Cytron'86]

Survey of Dependence Tests

gcd test

separability test (gcd test for special case, exact)

Banerjee-Wolfe test [Banerjee'88] rational solution in ItS

Delta-test [Goff/Kennedy/Tseng'91]

Power test [Wolfe/Tseng'91]

Simple Loop Residue test [Maydan/Hennessy/Lam'91]

Fourier-Motzkin Elimination [Maydan/Hennessy/Lam'91]

Omega test [Pugh/Wonnacott'92]

Loop Transformations and Parallelization

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Loop Optimizations – General Issues

- Move loop invariant computations out of loops
- Modify the order of iterations or parts thereof

Goals:

- Improve data access locality
- Faster execution
- Reduce loop control overhead
- Enhance possibilities for loop parallelization or vectorization

Only transformations that preserve the program semantics (its input/output behavior) are admissible

- Conservative (static) criterium: preserve data dependences
- Need data dependence analysis for loops (→ DF00100)

Some important loop transformations

- Loop normalization
- Loop parallelization
- Loop invariant code hoisting
- Loop interchange
- Loop fusion vs. Loop distribution / fission
- Strip-mining / loop tiling / blocking vs. Loop linearization
- Loop unrolling, unroll-and-jam
- Loop peeling
- Index set splitting, Loop unswitching
- Scalar replacement, Scalar expansion
- Later: Software pipelining
- More: Cycle shrinking, Loop skewing, ...

Loop Invariant Code Hoisting

- **Move loop invariant code out of the loop**
  - Compilers can do this automatically *if* they can statically find out what code is loop invariant
  - Example:

```
for (i=0;  i<10;  i++)
    a[i] = b[i]  + c / d;
```

⇒

```
tmp = c / d;
for (i=0;  i<10;  i++)
    a[i] = b[i]  + tmp;
```

Loop Unrolling

- **Loop unrolling**
  - Can be enforced with compiler options e.g. –funroll=2
  - Example:

```
for (i=0; i<50; i++) {
    a[i] = b[i];
}
```

Unroll by 2:

```
for (i =0; i<50; i+=2) {
    a[i] = b[i];
    a[i+1] = b[i+1];
}
```

☺ Reduces loop overhead (total # comparisons, branches, increments)

☺ Longer loop body may enable further local optimizations
(e.g. common subexpression elimination,
register allocation, instruction scheduling,
using SIMD instructions)

☹ longer code

→ Exercise: Formulate the unrolling rule for statically unknown upper loop limit

Loop Unrolling

for i **from** 1 **to** 100 **do**
$$a[i] \leftarrow a[i] + b[i]$$
od

unroll by 4:

for i **from** 1 **to** 100 **step** 4 **do**
$$a[i] \leftarrow a[i] + b[i]$$
$$a[i+1] \leftarrow a[i+1] + b[i+1]$$
$$a[i+2] \leftarrow a[i+2] + b[i+2]$$
$$a[i+3] \leftarrow a[i+3] + b[i+3]$$
od

+ less overhead per useful operation
+ longer basic blocks for local optimizations
  (local CSE, local reg.-allocation, local scheduling, SW pipelining)

– longer code

Loop Unrolling with Unknown Upper Bound

for i **from** 1 **to** N **do**
$\quad a[i] \leftarrow a[i] + b[i]$
od

unroll by 4:

$i \leftarrow 1$
while $i+3 < N$ **do**
$\quad a[i] \leftarrow a[i] + b[i]$
$\quad a[i+1] \leftarrow a[i+1] + b[i+1]$
$\quad a[i+2] \leftarrow a[i+2] + b[i+2]$
$\quad a[i+3] \leftarrow a[i+3] + b[i+3]$
$\quad i \leftarrow i + 4$
od
while $i < N$ **do**
$\quad a[i] \leftarrow a[i] + b[i]$
$\quad i \leftarrow i + 1$
od

used e.g. in BLAS

Loop Unroll-And-Jam

unroll the outer loop

and fuse the resulting inner loops:

for i **from** 1 **to** N **do**
  for j **from** 1 **to** N **do**
    $a[i] \leftarrow a[i] + b[j]$
  od
od

unroll&jam:

for i **from** 1 **to** N **step** 2 **do**
  for j **from** 1 **to** N **do**
    $a[i] \leftarrow a[i] + b[j]$
    $a[i+1] \leftarrow a[i+1] + b[j]$
  od
od

The same conditions as for loop interchange (for the two innermost loops after the unrolling step) must hold (for a formal treatment see [Allen/Kennedy'02, Ch. 8.4.1]).

+ increases reuse in inner loop
+ less overhead

Loop Peeling

remove the first (or last) iteration of the loop

and clone the loop body for that iteration.

for i **from** 1 **to** N **do**
    $a[i] \leftarrow (x+y)*b[i]$
od

peel first iteration:

if $N \geq 1$ **then**
    $a[1] \leftarrow (x+y)*b[1]$
    for i **from** 2 **to** N **do**
        $a[i] \leftarrow (x+y)*b[i]$
    od
fi

(Test on trip count can be removed if $N \geq 1$ is statically known.)

+ can enable loop fusion

+ may extract conditionals handling boundary cases from the loop

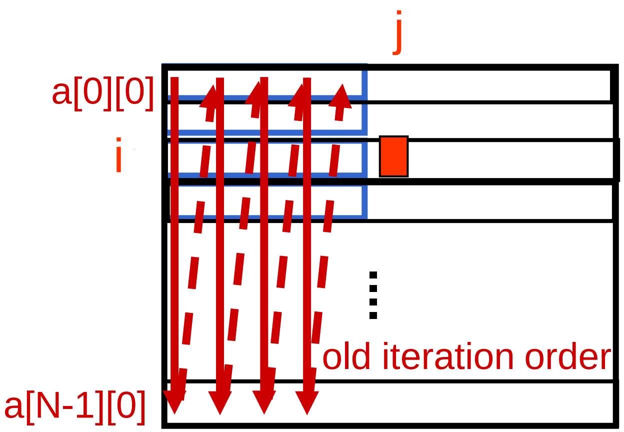
– longer code

Loop Interchange (1)

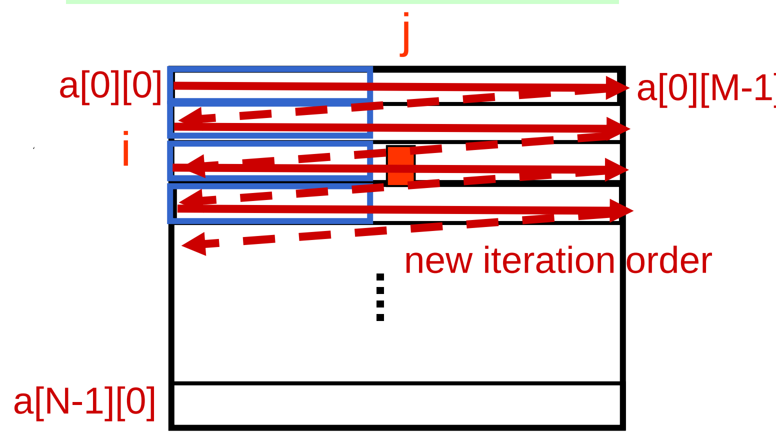
- For properly nested loops
  (statements in innermost loop body only)
  - Example 1:

```
for (j=0;  j<M;  j++)
   for (i=0;  i<N;  i++)
      a[ i ][ j ] = 0.0 ;
```

```
for (i=0;  i<N;  i++)
   for (j=0;  j<M;  j++)
      a[ i ][ j ] = 0.0 ;
```



row-wise storage of 2D-arrays in C, Java



  - Can improve data access locality in memory hierarchy
    (fewer cache misses / page faults)
  - Can help with subsequent vectorization of innermost loops

Recall: Loop-Carried Data Dependences

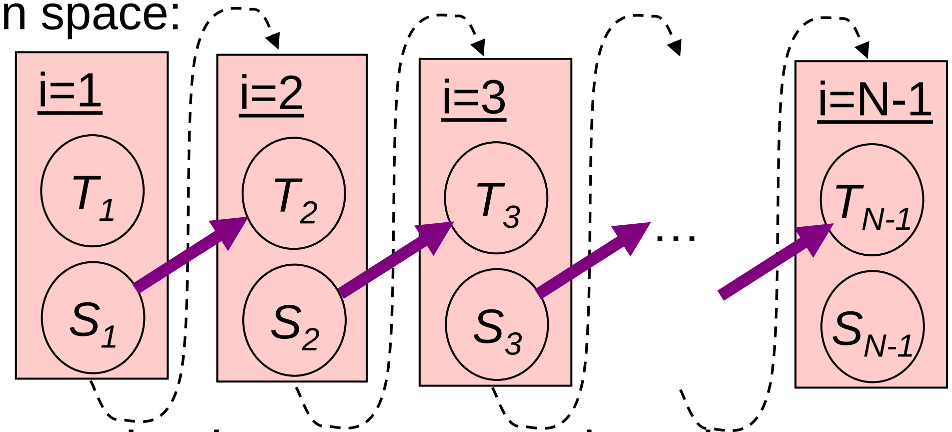
- Recall: **Data dependence** *S* → *T*, if operation *S* *may* execute (dynamically) before operation *T* and both *may* access the same memory location and at least one of these accesses is a write
  - In general, only a conservative over-estimation can be determined statically.

S: z = … ;
…
T: … = ..z.. ;

- Data dependence *S*→*T* is called ***loop carried*** by a loop *L* if the data dependence *S*→*T* may exist for instances of *S* and *T* in different iterations of *L*.
  - Example:

```
L:  for (i=1; i<N; i++) {
T_i:      …  =  x[ i-1 ];
S_i:      x[ i ] = …;
    }
```

Iteration space:



→ partial order between the operation instances resp. iterations

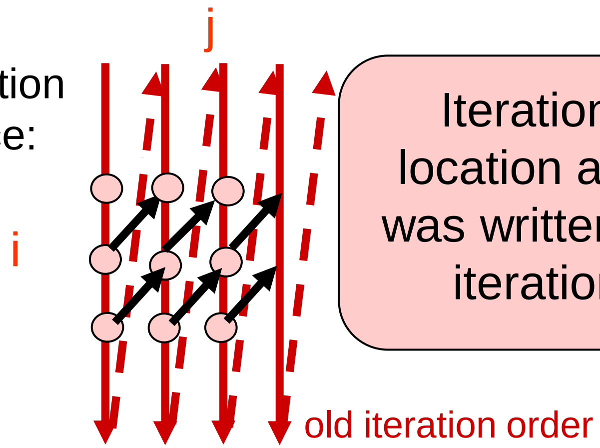
Loop Interchange (2)

- **Be careful** with loop carried data dependences!
  - Example 2:

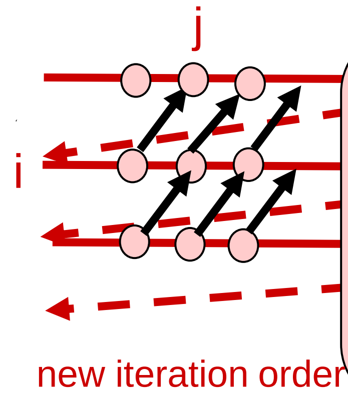
```
for (j=1;  j<M;  j++)
   for (i=0;  i<N;  i++)
      a[i][j] =…a[i+1][j-1]…;
```

```
for (i=0;  i<N;  i++)
   for (j=1;  j<M;  j++)
      a[i][j] =…a[i+1][j-1]…;
```

Iteration space:



Iteration (*j*,*i*) reads location a[*i+1*][*j-1*] that was written in an <u>earlier</u> iteration, (*i-1*,*j+1*)



Iteration (*i*,*j*) reads location a[*i+1*][*j-1*], that will be over-written in a <u>later</u> iteration (*i+1*,*j-1*)

  - Interchanging the loop headers would violate the partial iteration order given by the data dependences

Loop Interchange (3)

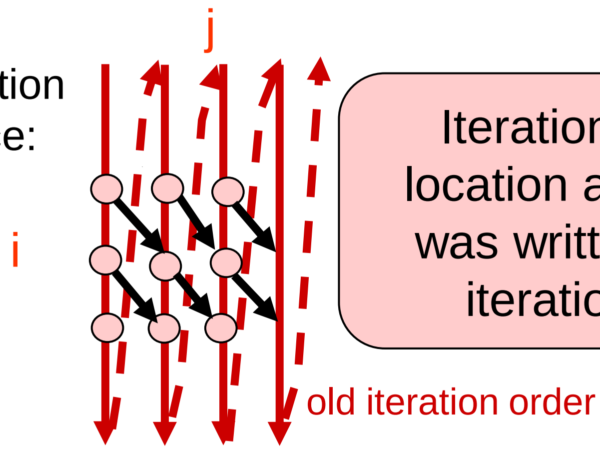
- **Be careful** with loop-carried data dependences!
  - Example 3:

```
for (j=1;  j<M;  j++)
   for (i=1;  i<N;  i++)
      a[i][j] =…a[i-1][j-1]...;
```

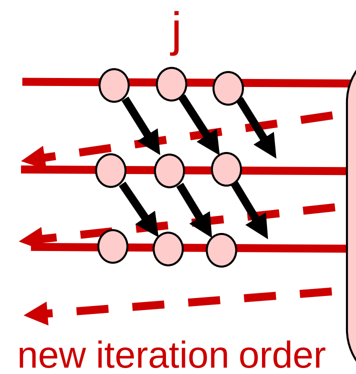
OK →

```
for (i=1;  i<N;  i++)
   for (j=1;  j<M;  j++)
      a[i][j] =…a[i-1][j-1]…;
```

Iteration space:



Iteration (*j,i*) reads location a[*i-1*][*j-1*] that was written in <u>earlier</u> iteration (*j-1,i-1*)



Iteration (*i,j*) reads location a[*i-1*][*j-1*] that was written in <u>earlier</u> iteration (*i-1,j-1*)

  - Generally: Interchanging loop headers is only admissible if loop-carried dependences have the <u>same direction</u> for all loops in the loop nest (all directed along or all against the iteration order)

Loop Fusion

- Merge subsequent loops with same header
  - Safe if neither loop carries a (backward) dependence
  - Example:

```
for (i=0;  i<N;  i++)
   a[ i ] = … ;
for (i=0;  i<N;  i++)
   … = … a[ i ] … ;
```

For N sufficiently large, a[*i*] will no longer be in the cache at this time

⇒

```
for (i= 0;  i<N;  i++) {
    a[ i ] = … ;
    … = … a[ i ] … ;
}
```

OK – Read of a[*i*] still after write of a[*i*], for all *i*

☺ Can improve data access locality and reduces number of branches

Loop Fusion
– Index variable name does not matter

for i **from** 1 **to** N **do**
  $c[i] \leftarrow a[i] + b[i]$
od

for j **from** 1 **to** N **do**
  $d[j] \leftarrow a[j] * e[j]$
od

fuse:

for i **from** 1 **to** N **do**
  $c[i] \leftarrow a[i] + b[i]$
  $d[i] \leftarrow a[i] * e[i]$
od

For array a large enough,
$a[i]$ will no longer be cached.

find second $a[i]$ in the cache
or even in a register

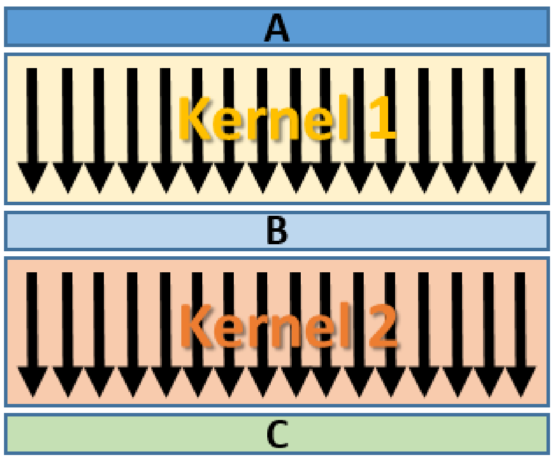
$j \leftarrow$ N (if downwards exposed)

Safe if neither loop carries a (backward) dependence.

+ locality: can convert inter-loop reuse to intra-loop reuse
+ larger basic blocks
+ reduce loop overhead

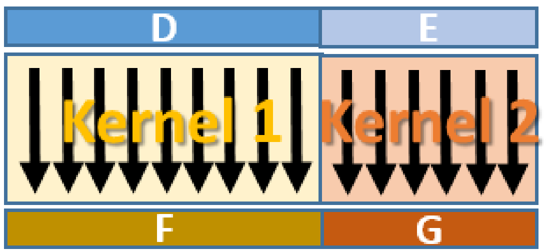
Special Case: Kernel Fusion for GPU

Serial Kernel Fusion




```
// start N1=N2 threads
{
    code_kernel1
    code_kernel2
}
```

Parallel Kernel Fusion



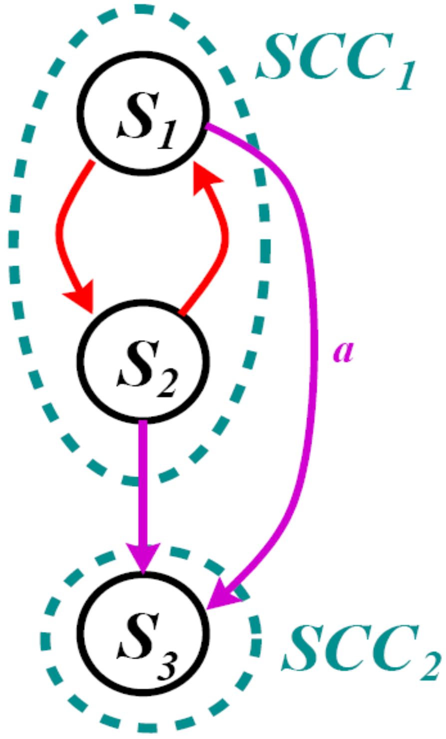

```
// start N1+N2 threads
{
  if (thread_idx < N1)
        code_kernel1
  else
        code_kernel2
}
```

Loop Distribution (a.k.a. Loop Fission)

```
    for (i=1; i<n; i++) {
S1:     a[i+1] = b[i-1] + c[i];
S2:     b[i]   = a[i] * k;
S3:     c[i]   = b[i] - 1;
    }
```

Loop distribution

```
    for (i=1; i<n; i++) {
S1:     a[i+1] = b[i-1] + c[i];
S2:     b[i]   = a[i] * k;
    }
    for (i=1; i<n; i++)
S3:     c[i]   = b[i] - 1;
```



Safe if all statements forming a SCC in the dependence graph end up in the same loop.

Forward (loop-carried) dep's are ok, but keep topological order.

\+ often enables vectorization; better cache utilization of each loop.

Loop Iteration Reordering

A transformation that reorders the iterations of a level-k-loop,
without making any other changes,
is valid if the loop carries no dependence.

Example:

```
   for (i=1; i<n; i++)
     for (j=1; j<m; j++)
       for (k=1; k<r; k++)
S:       a[i][j][k] = ... a[i][j-1][k] ...          (=,<,=)
```

j-loop carries a dependence, its iteration order must be preserved

Loop Parallelization

A transformation that reorders the iterations of a level-k-loop,
without making any other changes,
is valid if the loop carries no dependence.

Example:

```
   for (i=1; i<n; i++)
      for (j=1; j<m; j++)
         for (k=1; k<r; k++)
S:          a[i][j][k] = ... a[i][j-1][k] ...          (=,<,=)
```

j-loop carries a dependence, its iteration order must be preserved

It is valid to convert a sequential loop to a parallel loop
if it does not carry a dependence.

Example:

```
   for (i=1; i<n; i++)                            forall ( i, 1, n, p )
S:   b[i] = 2 * c[i];                               b[i] = 2 * c[i];
```

Loop parallelization

Principle: Parallelize outermost loop(s), vectorize innermost loop(s)

Remark on Loop Parallelization

- Introducing temporary copies of arrays can remove some antidependences to enable automatic loop parallelization

- Example:

```
for (i=0; i<n; i++)
    a[i] = a[i] + a[i+1];
```

- The loop-carried dependence can be eliminated:

```
for (i=0; i<n; i++)
    aold[i+1] = a[i+1];
for (i=0; i<n; i++)
    a[i] = a[i] + aold[i+1];
```

Parallelizable loop

Parallelizable loop

Strip Mining / Loop Blocking

```
for (i=0; i<n; i++)
      a[i] = b[i] + c[i];
```

Loop blocking with block size *s*

```
for (ii=0; ii<n; ii+=s)          // loop over blocks
   for (i=ii; i<min(ii+s,n); i++)  // loop within block
      a[i] = b[i] + c[i];
```

Reverse transformation: Loop linearization

Loop (Nest) Tiling

```
for (i=0; i<n; i++)
    for (j=0; j<m; j++)
        a[i][j] = b[i][j] + c[j][i];
```

Loop nest tiling with tile size *s* x *s* - **Step 1**: loop blocking

```
for (ii=0; ii<n; ii+=s)      // loop over blocks
  for (i=ii; i<min(ii+s,n); i++)  // loop within block
    for (jj=0; jj<m; jj+=s) // loop over blocks
      for (j=jj; j<min(jj+s,m); j++) // loop within blk
         a[i][j] = b[i][j] + c[j][i];
```

Loop (Nest) Tiling

```
for (i=0; i<n; i++)
    for (j=0; j<m; j++)
        a[i][j] = b[i][j] + c[j][i];
```

Loop nest tiling with tile size *s* x *s* **- Step 2:** Loop interchange

```
for (ii=0; ii<n; ii+=s)      // loop over blocks
  for (jj=0; jj<m; jj+=s)    // loop over blocks
    for (i=ii; i<min(ii+s,n); i++) // loop within block
      for (j=jj; j<min(jj+s,m); j++) // loop within blk
        a[i][j] = b[i][j] + c[j][i];
```

Tiling = **loop blocking** for *multiple* loop headers in a loop nest
+ loop interchange
→ loops scanning a tile become innermost loops

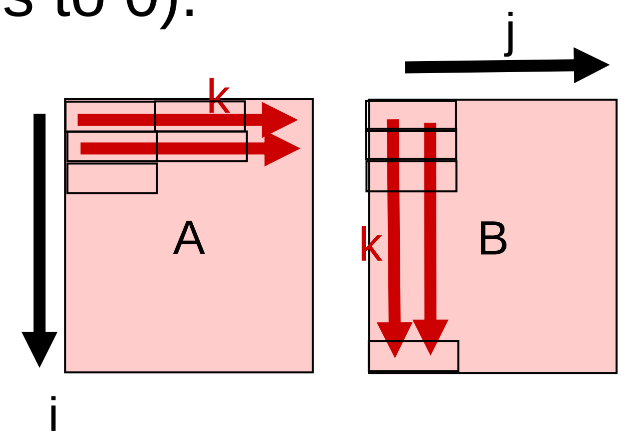
Goal: increase locality; support vectorization (vector registers)

Tiled Matrix-Matrix Multiplication (1)

- Matrix-Matrix multiplication $C = A \times B$
  here for square ($n \times n$) matrices $C, A, B,$ with n large ($\sim 10^3$):
  - $C_{ij} = \Sigma_{k=1..n} \; A_{ik} \; B_{kj}$ for all $i, j = 1...n$

- Standard algorithm for Matrix-Matrix multiplication
  (here without the initialization of C-entries to 0):

```
for (i=0; i<n; i++)
   for (j=0;  j<n;  j++)
      for (k=0;  k<n;  k++)
         C[i][j] += A[i][k] * B[k][j];
```

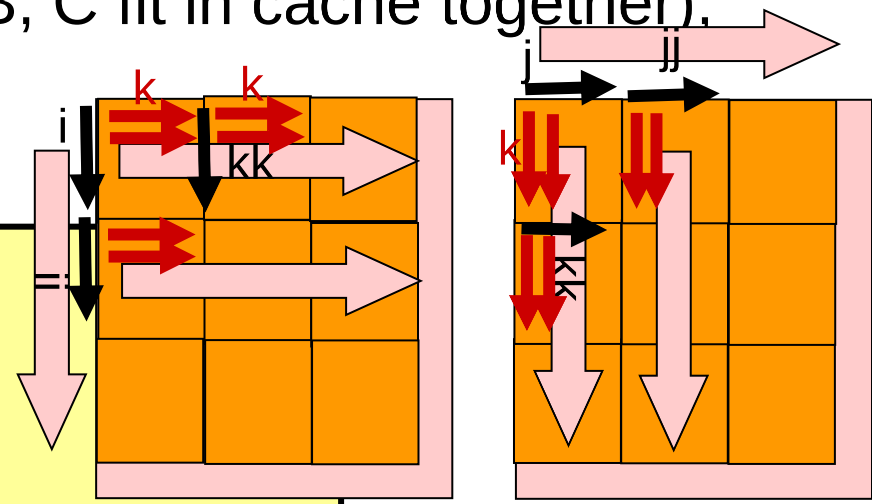


Good spatial locality on A, C

Bad spatial locality on B
(many capacity misses)

Tiled Matrix-Matrix Multiplication (2)

- Block each loop by block size S
(choose S so that a block of A, B, C fit in cache together),
then interchange loops



- Code after tiling:

```
for (ii=0; ii<n; ii+=S)
    for (jj=0;  jj<n;  jj+=S)
        for (kk=0;  kk<n;  kk+=S)
            for (i=ii;  i < ii+S;  i++)
                for (j=jj;  j < jj+S;  j++)
                    for (k=kk;  k < kk+S;  k++)
                        C[i][j] += A[i][k] * B[k][j];
```

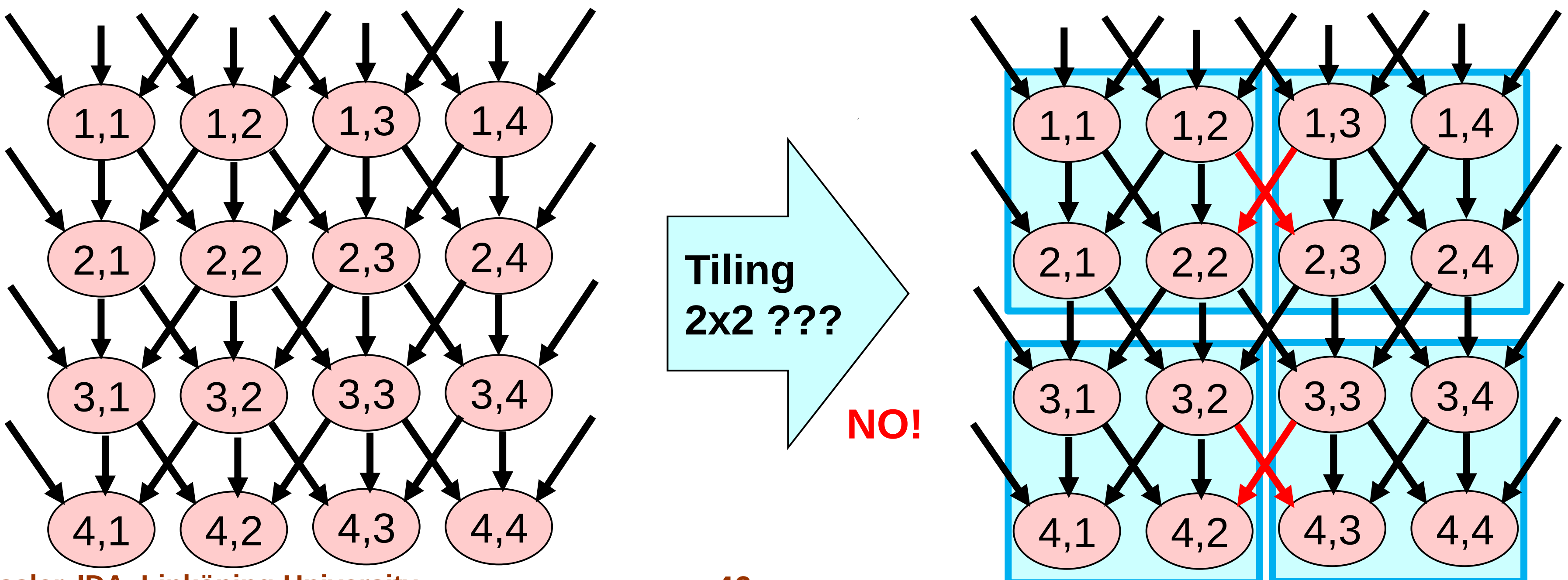
Good spatial locality
for A, B and C

Loop (Nest) Tiling (cont.)

- Beware: Tiling is not always semantics-preserving
  - Dependences could lead to unschedulable code

<u>Example:</u>

```
        for i = 1, ..., 4
             for j = 1, ..., 4
S(i,j):             A[i][j] = x*A[i-1][j-1] + y*A[i-1][j] + z*A[i-1][j+1];
```



Remark on Locality Transformations

- An alternative can be to change the data layout rather than the control structure of the program
  - **Example:** Store matrix B in transposed form, or, if necessary, consider transposing it, which may pay off over several subsequent computations
    - Finding the best layout for all multidimensional arrays is a NP-complete optimization problem [Mace, 1988]
  - **Example:** Recursive array layouts that preserve locality
    - Morton-order layout
    - Hierarchically tiled arrays
- In the best case, can make computations *cache-oblivious*
  - Performance largely independent of cache size

- **Further example**: AOS vs. SOA layout for images on CPU/GPU

Loop Nest Flattening / Linearization

Flattens a multidimensional iteration space to a linear space:

$$\begin{array}{l}\textbf{for } i \textbf{ from } 0 \textbf{ to } n-1 \textbf{ do}\\ \quad \textbf{for } j \textbf{ from } 0 \textbf{ to } m-1 \textbf{ do}\\ \quad\quad \text{iteration}(i,j)\\ \quad \textbf{od}\\ \textbf{od}\end{array}$$

linearize:

$$\begin{array}{l}\textbf{for } k \textbf{ from } 0 \textbf{ to } m \cdot n-1 \textbf{ do}\\ \quad i \leftarrow k \,/\, m\\ \quad j \leftarrow k \,\%\, m\\ \quad \text{iteration}(i,j)\\ \textbf{od}\end{array}$$

+ larger iteration space, better for scheduling / load balancing

– overhead to reconstruct original iteration variables
may be reduced by using *induction variables* i, j
that are updated by accumulating additions instead of div and mod

Index Set Splitting

Divide the *iteration space* into two portions.

for i **from** 1 **to** 100 **do**
  $a[i] \leftarrow b[i] + c[i]$
  if $i > 10$ **then**
    $d[i] \leftarrow a[i] + a[i-10]$
  fi
od

split after 10:

for i **from** 1 **to** 10 **do**
  $a[i] \leftarrow b[i] + c[i]$
od
for i **from** 11 **to** 100 **do**
  $a[i] \leftarrow b[i] + c[i]$
  $d[i] \leftarrow a[i] + a[i-10]$
od

+ removes condition evaluation in every iteration
+ factors out the parallelizable set of iterations

– longer code

Loop Unswitching

```
for i from 1 to 100 do
    a[i] ← a[i] + b[i]
    if expression then
        d[i] ← 0
    fi
od
```

unswitch:

```
if expression then
    for i from 1 to 100 do
        a[i] ← a[i] + b[i]
        d[i] ← 0
    od
else
    for i from 1 to 100 do
        a[i] ← a[i] + b[i]
    od
fi
```

+ hoist loop-invariant control flow out of loop nest
+ no tests, no branches in loop body
  → larger basic blocks (see above), simpler software pipelining

– longer code

Scalar Expansion / Array Privatization

promote a scalar temporary to an array to break a dependence cycle

for i **from** 1 **to** N **do**
  $t \leftarrow a[i] + b[j]$
  $c[i] \leftarrow t + 1$
od

expand scalar t:

if $N \geq 1$
  allocate $t'[1..N]$
  for i **from** 1 **to** N **do**
    $t'[i] \leftarrow a[i] + b[j]$
    $c[i] \leftarrow t'[i] + 1$
  od
  $t \leftarrow t'[N]$ // *if t live on exit*
fi

+ removes the loop-carried antidependence due to t

  $\rightarrow$ can now parallelize the loop!

- needs more array space

Loop must be countable, scalar must not have upward exposed uses.

May also be done conceptually only, to enable parallelization:
just create one private copy of t for every processor = array privatization

Idiom recognition and algorithm replacement

Traditional loop parallelization fails for loop-carried dep. with distance 1:

```
S0:  s = 0;
     for (i=1; i<n; i++)
S1:    s = s + a[i];

S2:  a[0] = c[0];
     for (i=1; i<n; i++)
S3:    a[i] = a[i-1] * b[i] + c[i];
```

C. Kessler: Pattern-driven automatic parallelization. *Scientific Programming*, 1996

A. Shafiee-Sarvestani, E. Hansson, C. Kessler: Extensible recognition of algorithmic patterns in DSP programs for automatic parallelization. *Int. J. on Parallel Programming*, 2013.

↓ Idiom recognition (pattern matching)

```
S1': s = VSUM( a[1:n-1], 0 );

S3': a[0:n-1] = FOLR( b[1:n-1], c[0:n-1], mul, add );
```

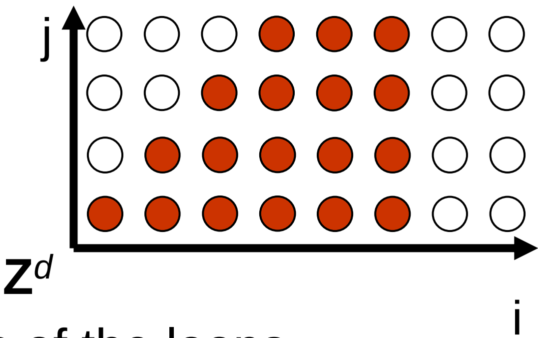
↓ Algorithm replacement

```
S1'': s = par_sum( a, 0, n, 0 );
```

Polyhedral / Polytope Model

```
for i = 1 to N
    for j = min(i,M) to M
        loopbody( i, j )
```

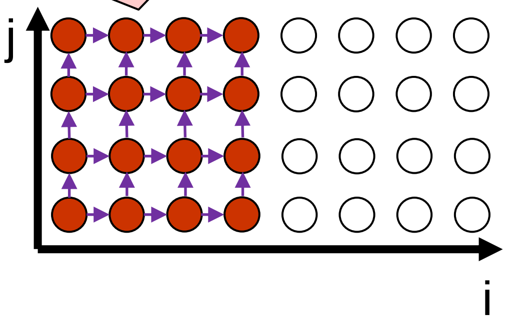
- Researched since late 1980s (with earlier roots), still active (see e.g. IMPACT workshop series), tensor computations
- Compact representation of the **loop nest iteration space** of *d* perfectly nested loops as the points of a *polytope* (*polyhedron*) in $\mathbf{Z}^d$
  - Usually, loop normalization to obtain stride +1
  - E.g. in 2D: rectangular, triangular, trapezoidal, etc.



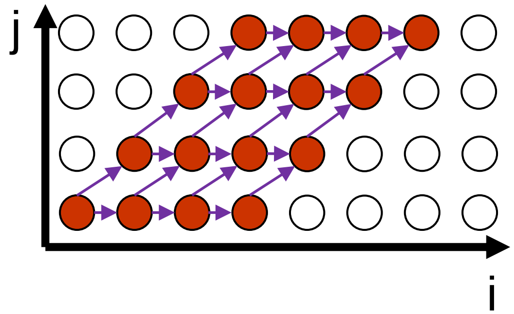
- Loop bounds must be **affine** (linear) functions of the indexes of outer loops (or constant)
  - The **polytope** is the intersection of halfspaces over $\mathbf{Z}^d$
  - The faces of the polytope are defined by the bounds of the loops
- Can apply described loop transformations as dependences allow
  - Can often be described as unimodular linear mappings
- **Parallelism** and scheduling options can be determined statically
  - constrained by the data dependences
- **Schedule** = space-time mapping of iterations to parallel processors and time axis must be affine.
- **Code generator** (eg. cloog, MLIR lowering) generates code (nest of *d* for-loops) that scans the polyhedron, given index bound parameters and a schedule

Polyhedral Example: Loop Nest Skewing and Parallelization

```
for i = 1 to N
   for j = 1 to M
      a[i,j] = f( a[i-1,j], a[i, j-1] )
```

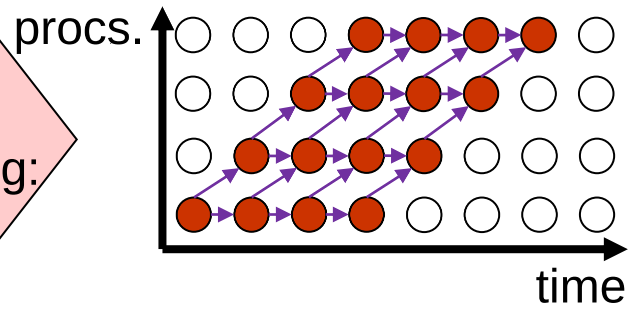


skewing:



(assuming here for simplicity that we have procs = N parallel processing units to use. If not, apply strip mining / tiling ...)

mapping/ scheduling:



generate HIR/src code:

```
forall proc = 1 to N
   for time = min(proc, N) to max(M+proc, M+N-1)
      a[i,j] = f( a[time-1, proc-1], a[time-1, proc] )
```

Concluding Remarks

Limits of Static Analyzability

Outlook: Runtime Analysis and Parallelization

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Remark on static analyzability (1)

- Static dependence information is always a (safe) overapproximation of the real (run-time) dependences
  - Finding out the real ones exactly is statically undecidable!
  - If in doubt, a dependence must be assumed
    → may prevent some optimizations or parallelization
- One main reason for imprecision is **aliasing**, i.e. the program may have several ways to refer to the same memory location
  - Example: Pointer aliasing

```
void  mergesort ( int *a, int n )
{  …
   mergesort ( a,  n/2 );
   mergesort ( a + n/2, n-n/2 );
   …
}
```

How could a static analysis tool (e.g., compiler) know that the two recursive calls read and write disjoint subarrays of a?

Remark on static analyzability (2)

- Static dependence information is always a (safe) overapproximation of the real (run-time) dependences
  - Finding out the latter exactly is statically undecidable!
  - If in doubt, a dependence must be assumed
    → may prevent some optimizations or parallelization
- Another reason for imprecision are **statically unknown values** that imply whether a dependence exists or not
  - Example: Unknown dependence distance

```
// value of K statically unknown
for ( i=0; i<N; i++ )
{  …
   S:   a[i] = a[i] + a[K];
   …
}
```

Loop-carried dependence if K < N.
Otherwise, the loop is parallelizable.

Outlook: Runtime Parallelization

Sometimes parallelizability cannot be decided statically.

if is_parallelizable(...)
    forall i **in** [0..n-1] **do**   *// parallel version of the loop*
        iteration(i);
    od
else
    for i **from** 0 **to** $n-1$ **do**   *// sequential version of the loop*
        iteration(i);
    od
fi

The runtime dependence test is_parallelizable(...) itself may partially run in parallel.

Run-Time Parallelization

Goal of run-time parallelization

- Typical target: **irregular loops**

  for (i=0; i<n; i++)
  a[i] = *f* (a[*g*(i)], a[*h*(i)], ...);

  - Array index expressions *g, h...* depend on run-time data
  - Iterations cannot be statically proved independent (and not either dependent with distance +1)

- **Principle:**
  At runtime, inspect *g, h* ... to find out the real dependences and compute a schedule for partially parallel execution
  - Can also be combined with speculative parallelization

Overview

- **Run-time parallelization of irregular loops**
  - DOACROSS parallelization
  - Inspector-Executor Technique (shared memory)
  - Inspector-Executor Technique (message passing) *
  - Privatizing DOALL Test *
- **Speculative run-time parallelization of irregular loops ***
  - LRPD Test *
- **General Thread-Level Speculation**
  - Hardware support *

* = not covered in this lecture. See the references.

DOACROSS Parallelization

- Useful if loop-carried dependence distances are unknown, but often > 1
- Allow independent subsequent loop iterations to overlap
- Bilateral synchronization between really-dependent iterations

Example:

```
for ( i=0; i<n; i++)
        a[i]  =  f ( a[ g(i) ], ... );
```

```
sh float aold[n];
sh flag done[n];   // flag (semaphore) array
forall i in 0..n-1   {   // spawn n threads, one per iteration
    done[i] = 0;
    aold[i] = a[i];    // create a copy
}
forall i in 0..n-1  {   // spawn n threads, one per iteration
    if (g(i) < i)   wait until done[ g(i) ] );
                    a[i]  =  f ( a[ g(i) ], ... );
                    set( done[i] );
    else
                    a[i]  =  f ( aold[ g(i) ], ... );  set done[i];
}
```

Inspector-Executor Technique (1)

- Compiler generates 2 pieces of customized code for such loops:

- **Inspector**
  - calculates values of index expression
    by simulating whole loop execution
    - typically, based on sequential version of the source loop
      (some computations could be left out)
  - computes implicitly the real iteration dependence graph
  - computes a **parallel schedule** as (greedy) wavefront traversal of the iteration dependence graph in topological order
    - all iterations in same wavefront are independent
    - schedule **depth** = #wavefronts = critical path length

- **Executor**
  - follows this schedule to execute the loop

Inspector-Executor Technique (2)

- **Source loop:**

```
for ( i=0; i<n; i++)
        a[i]  =  f ( a[ g(i) ], a[ h(i) ], ... );
```

- **Inspector:**

```
int wf[n];  // wavefront indices
int depth = 0;
for (i=0; i<n; i++)
   wf[i] = 0;   // init.
for (i=0; i<n; i++) {
   wf[i] = max ( wf[ g(i) ], wf[ h(i) ], ... ) + 1;
   depth = max ( depth, wf[i] );
}
```

- Inspector considers only flow dependences (RAW), anti- and output dependences to be preserved by executor

Inspector-Executor Technique (3)

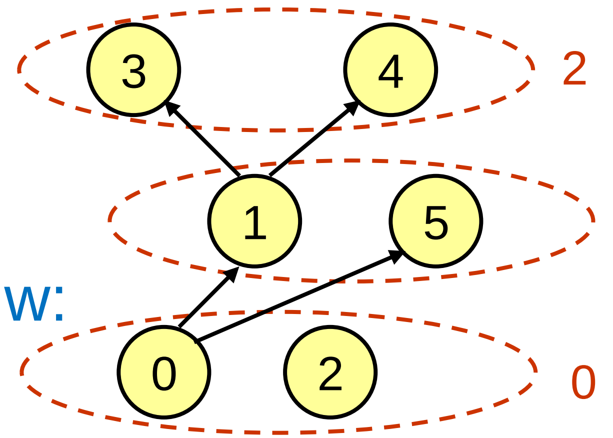
- **Example**:

```
for (i=0; i<n; i++)
     a[i] = ... a[ g(i) ] ...;
```

i	0	1	2	3	4	5
g(i)	2	0	2	1	1	0
wf[i]	0	1	0	2	2	1
g(i)<i ?	no	yes	no	yes	yes	yes

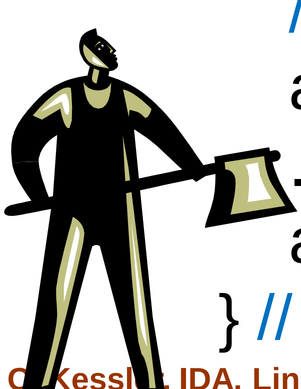
- **Executor**:

```
float aold[n];  // buffer array
aold[1:n] = a[1:n];
for (w=0; w<depth; w++)
   forall (i in {0..n-1}: wf[i] == w)  {
        // start task/thread where wf[i] == w:
        a1 = (g(i) < i)? a[g(i)] : aold[g(i)];
        ...  // similarly, a2 for h etc.
        a[i] =  f ( a1, a2, ... );
   } // wait for all threads of round w
```



iteration (flow) dependence graph

(depth=3)



Inspector-Executor Technique (4)

Problem: Inspector remains sequential – no speedup

Solution approaches:

- Re-use schedule over subsequent iterations of an outer loop if access pattern does not change
  - amortizes inspector overhead across repeated executions
- Parallelize the inspector using doacross parallelization [Saltz,Mirchandaney'91]
- Parallelize the inspector using sectioning [Leung/Zahorjan'91]
  - compute processor-local wavefronts in parallel, concatenate
  - trade-off schedule quality (depth) vs. inspector speed
  - Parallelize the inspector using bootstrapping [Leung/Z.'91]
  - Start with suboptimal schedule by sectioning, use this to execute the inspector → refined schedule

Some references on Dependence Analysis, Loop optimizations and Transformations

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- M. Wolfe: *High-Performance Compilers for Parallel Computing*. Addison-Wesley, 1996.
- R. Allen, K. Kennedy: *Optimizing Compilers for Modern Architectures*. Morgan Kaufmann, 2002.

Idiom recognition and algorithm replacement:

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Questions?

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Frameworks

- Polly
- Cloog
- PluTo polyhedral transformation framework:
  An automatic parallelizer and locality optimizer for affine loop nests
  http://pluto-compiler.sourceforge.net/

Polyhedral Compilation Frameworks

- Closely related to (parametric) integer programming
  - PIPS, PIPlib
  - Paul Feautrier: Dataflow Analysis of Array and Scalar References. International Journal of Parallel Programming, 1991
- and many others

More recent work e.g.

- Polly for LLVM: https://polly.llvm.org/
- PluTo
  - U. Bondhugula, PhD thesis, 2008: https://www.csa.iisc.ac.in/~udayb/publications/uday-thesis.pdf
- Cloog
  - for code generation (scanning a polyhedron, given iteration domain bounds and a schedule)
  - http://www.cloog.org
- Polybench polyhedral benchmark suite
- Annual IMPACT workshop series at HiPEAC conference

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